THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XX OCTOBER 1904 NUMBER 3

I. A METHOD OF MEASUREMENT AND REDUCTION OF SPECTROGRAMS FOR THE DETERMINATION OF RADIAL VELOCITIES.

II. APPLICATION TO A STUDY OF THE VARIABLE STAR *W SAGITTARII*.[1]

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INTRODUCTION.

BEGINNING thirty-five years ago, when Huggins and Vogel first made application of Doppler's principle to the determination of radial velocities of stars, the methods employed in this work have been developed, until at present an efficient and practically uniform system has been adopted in the various astrophysical observatories. In 1888, when the attempt was made at Potsdam to record photographically the displacement of lines in the stellar spectra, the problem was placed on a firm basis. Subsequently, through improvement in the construction of spectrographs, a more accurate knowledge of the wave-lengths of spectral lines, and signal advances in the methods of measurement and reduction of the spectrograms, the results have attained their present degree of accuracy. The probable error of 22 km per

[1] Dissertation in partial fulfilment of the requirements for the degree of doctor of philosophy in the University of California. Also to appear as a *Bulletin* of the Lick Observatory.

second for an average case of velocity determination with the older methods has been reduced to a few tenths of a kilometer at the present time.

Flexure and temperature-change during exposure may be said to play no part in the performance of the modern properly equipped spectrograph, and other observational sources of error are well under control. Undoubtedly, the spectrograms of today contain information regarding the velocities of celestial light-sources more precise than our present methods of measurement and reduction are capable of bringing out. In these methods there exist certain recognized sources of accidental and systematic error which must be eliminated if the coveted hundreth of a kilometer can ever have any significance.

Evidences of the extent of uncertainty which characterizes the spectrographic results of the last eight years can be obtained by comparison of determinations of velocity from the same star with different instruments. In earlier work strong systematic differences were encountered. In measures of the star *ζ Herculis*[1] the difference Campbell (5 plates)–Bélopolsky (5 plates) $= -3.5$ km. Two more plates by Bélopolsky greatly diminish this difference, but increase the probable error of his mean so signally that their inclusion is hardly consistent. For *ζ Geminorum*[2] the residual Campbell (44 plates) – Bélopolsky (15 plates) $= -6$ km. For *α Persei*[3] the residual Campbell (4 plates) – Vogel (13 plates) $= -0.8$ km. In the case of *Polaris*,[4] Hartmann finds: Campbell – Bélopolsky $= -3$ km, and for Campbell–Frost $= +2.2$ km. For *Capella*[5] the difference Campbell (31 plates) – Newall (23 plates) $= +3$ km. For purposes of this comparison the investigations of standard velocity stars have not progressed far enough as yet. But from observations made at Yerkes Observatory Frost and Adams conclude that "there would appear to be a slight tendency toward a systematic difference between our results and those of other observers in the direction of a larger positive, or a smaller negative, value for our velocities."[6] That any large systematic discrepancies will be brought out by these

[1] ASTROPHYSICAL JOURNAL, **8**, 157, 1898.
[2] *Ibid.*, **13**, 90, 1901.
[3] *Ibid.*, **13**, 320, 1901.
[4] *Ibid.*, **14**, 52, 1901.
[5] *Ibid.*, **12**, 251, 1900.
[6] *Ibid.*, **18**, 276, 1903.

results from standard stars is highly improbable. The instruments involved have been in use long enough to be thoroughly under control, and the choice of lines from the star and check-plates would naturally be governed by their performance in previous comparisons with the theoretical velocities of the Moon and planets. An uncertainty of at least several tenths of a kilometer, which, I think, is largely attributable to sources of error in the treatment of the plate, undoubtedly remains in spectrographic velocity determinations.

I. METHODS OF MEASUREMENT AND REDUCTION OF SPECTROGRAMS.

The plan of procedure that has been widely adopted, subject to slight variation in the case of different observers, may be briefly described. The spectrogram is measured twice, direct and reversed, on an engine supplied with a microscope of adjustable magnifying power, rigidly mounted over a plate-carriage which is moved by a delicate screw and micrometer system in one co-ordinate only, in the direction of extension of the spectrum. A set of measures includes the list of micrometer readings which correspond to the positions of the plate-carriage when the various spectral lines, both bright and dark, occupy in turn a given position with reference to some fixed point in the focal plane of the microscope. These readings, together with the laboratory determinations of the wave-lengths of the spectral lines employed for the measurements, constitute the data necessary for the determination of the radial velocity of the source emitting the light in terms of the accepted standards of velocity. From the micrometer readings and wave-lengths corresponding to not less than three known lines in the comparison or solar spectrum, the constants of the Hartmann–Cornu interpolation formula[1] are easily computed. By this formula micrometer readings are readily transformed into approximate wave-lengths, and *vice versa*. If, now, each plate is to be reduced independently, as at Yerkes Observatory,[2] the constants of the empirical interpolation curve are computed as above from three or more comparison lines. Corrections to this

[1] ASTROPHYSICAL JOURNAL, **8**, 218, 1898; *Astronomische Nachrichten*, **155**, 81–118, 1901.

[2] *Publications of the Yerkes Observatory*, **2**, 150, 1903; ASTROPHYSICAL JOURNAL, **15**, 22, 1902.

curve are determined by comparison of the adopted and interpolated wave-lengths, and are plotted as functions of the micrometer readings or the wave-lengths themselves. It then remains to compute by means of this corrected interpolation formula the wave-lengths corresponding to the micrometer settings on the lines of the spectrum to be measured. We thus obtain the actual wave-lengths of these lines as affected by the motion of their source relatively to the observer. From the known wave-lengths of these lines, occurring in the spectrum of a light-source at rest with reference to the observer, a simple application of Doppler's principle will yield the desired radial velocity of the star. If, on the other hand, the dispersions of one or more fundamental solar plates[1] be taken as standards, by measures on the comparison lines of a stellar spectrogram, the micrometer readings on the star lines themselves are reduced to the selected fundamental dispersion and may be compared directly with the computed readings of the same lines in the fundamental table. In point of accuracy, there seems to be no choice between these two methods, but the latter one is undoubtedly shorter. The sources of error, which are shared by both, I shall briefly review.

SOURCES OF ERROR IN THE REDUCTION AND MEASUREMENT OF SPECTROGRAMS.

a) *Change in film.*—The actual shape of the photographic film while the plate is being exposed is never reproduced, owing to distortions ascribable to temperature change and the processes of development and drying; but the effect of these distortions on velocity determinations is minute and intangible, especially when many well distributed comparison lines are employed.

b) *The measuring engine.*—(1) Errors in screw. Errors due to inequalities in the pitch of the micrometer screw, whether periodic or otherwise, are usually negligible in modern engines. They may, of course, be eliminated by application of small corrections to the measures, but are essentially inoperative in most velocity work.

(2) Temperature change during measures. The effect of temperature change on the measuring engine during measurements may play an important part unless considerable care is exercised. Meas-

[1] *Astronomische Nachrichten*, **155**, 101, 1901; ASTROPHYSICAL JOURNAL, **8**, 124, 1898.

ures should not be begun until the micrometer-head has been in contact with the hand for a short time, and during the measures the room temperature should be kept nearly constant. In my own work, after completing a plate I have tested the temperature shift by setting back on the lines first measured, and have invariably found some discrepancy. In one solar plate in particular, which had required considerable time for measurement, this shift amounted to about 0.003 mm, or the equivalent of seven kilometers' velocity. This led me to test the character of the shift by remeasurement of some of the remaining comparison lines on the plate. The plot of differences thus obtained approximated a straight line which gradually approached the original curve. Evidently the temperature-change was progressive and was entirely taken up in the first curve. Here, again, the advantage of an even distribution of star and comparison lines is apparent. Evidently, also, the "smoothing out" of curves of residuals from interpolation formulæ should not be practiced without due caution, for temperature-change during the measures may produce very appreciable distortions in such a plot.

c) *Subjective errors.*—Discrepancies arising from accidental errors of setting and from personal equation are among the most important to be considered. The former can be reduced only by long practice and the employment of a greater number of lines; the latter have been largely eliminated by the reversal of the spectrogram on the engine,[1] but it should be noted that in the reversal of a plate the spectrum is simultaneously inverted. Thus, in general, it would hardly be said that the effect of personal equation is entirely compensating in the two measures, for the appearance of a line after inversion might be so changed as seriously to interfere with the duplication of the conditions which existed before reversal. Further, in case of a straight-slit spectrograph the curvature[2] of the spectral lines would seriously interfere with the elimination of personal equation in this way.

d) *Errors due to the assumption of wave-lengths.*—(1) An instrumental defect. A source of error by no means inappreciable is found

[1] *Lick Observatory Bulletin* No. 15, and ASTROPHYSICAL JOURNAL, **15**, 208, 1902.

[2] Curved slits are used on the Mills spectrographs. They give straight spectral lines, which is a decided advantage.

in the relative displacement of comparison and absorption lines due to imperfect focal conditions which obtain outside of given narrow regions of the spectrum corresponding to the rays which pass through the prism-train at minimum deviation, or corresponding to the intersections of the focal curve with the plane surface of the photographic plate. The so-called wings, which seriously impair the definition of the outlying lines of the comparison spectrum, undoubtedly persist far into the region of apparently sharp focus, giving rise to errors in the measured positions of the bright lines. The corresponding displacements of the absorption lines of the stellar spectra, due to the same cause, are certainly much smaller, if at all appreciable. This difference is due to the fact that much light is concentrated in the bright lines, many of which are strongly exposed in order to bring out the fainter features of the comparison. Thus the wings, though they share a relatively small part of the light, may become quite as dense as the sharp part of the lines, for the density of the sharp part of the lines is but slightly affected by light after a certain time, while the wings continue to spread.

The nature of the effect of this source of error is difficult to predict, for it depends entirely upon the optical parts of the spectrograph and the character of the adjustments. If the focal curve of the camera lens is tangent to the plane of the surface of the photographic plate in the region of the spectrum corresponding to minimum deviation, the error arising would certainly differ from that which exists when the focal curve intersects the plate in two lines, though the character of the difference would be very uncertain. The results of a few investigations of actual measures with various instruments appear in the accompanying table. For each set of measures the means were formed of the velocity in kilometers as obtained from a given number of lines to the violet, to the red, and at minimum deviation. These means are indicated by V, R, and M, respectively, and the means of the differences between these quantities for an entire set of plates are given under the heads V–R, V–M, and R–M. The number of lines used on each plate in the formation of the means V, R, and M is given in another column, and the approximate range in wave-length between the means V and R appears under the heading "Range." The probable errors of the quantities determined are also added.

TABLE I.

Spectrographs	V–M	R–M	V–R	No. Lines on Each Plate	No. of Measures of Plates	Range
Bruce	....	....	+0.16km ±0.08	6	84	98μ
Pulkowa	....	....	−0.85 ±0.20	6	35	200μ
Mills	−1.76km ±0.16	+1.10km ±0.20	−0.66 ±0.14	30	20	180μ

The results for the Bruce and Pulkowa spectrographs were obtained from data recently published in connection with the velocity determinations of standard stars. The measures of Frost and Adams for each plate were reduced separately. The results for the Mills spectrograph were obtained from a set of measures of negatives of *Polaris*, which afforded better material than the data used for the other instruments. The range covered by the Yerkes measures is so small that no marked effect could be expected, but for the other instruments there seems to be a fairly consistent difference between measures of lines in different parts of the spectrum. These results may be masked by errors in wave-lengths employed, but the discrepancies are strong enough to call for further investigation.

(2) Physical differences. Aside from subjective errors of measurement, there is no greater source of uncertainty in the reduction of spectrograms than that which arises from the assumption of wave-lengths for the star and comparison lines from Rowland's solar tables. It is doubtful if the conditions of density and internal motion in the Sun and spark are sufficiently alike to warrant this procedure. But until better determinations of spark wave-lengths are available, there seems to be no alternative. A careful discussion of the matter by Frost and Adams[1] has led to the conclusion that

> In the present state of our knowledge we .... cannot say with any certainty how much our results are affected by the use of solar wave-lengths for our *Ti* lines; but presumably by an amount corresponding to less than 0.02 tenth-meters, or about 1.4 kilometers, and perhaps very much less.

Wave-lengths in star and Sun may differ by small amounts, due to

[1] *Publications of the Yerkes Observatory*, **2**, 155.

pressure, etc., but it seems practicable to assume agreement in the case of solar-type stars. In other cases the chances are favorable.

(3) Errors in Rowland's table. Rowland's determinations of solar wave-lengths are probably relatively accurate to the nearest hundredth of an Ångström unit. Their absolute value need be known only approximately for velocity work.

(4) Errors in practice. In actual performance some apparently good lines are found to yield consistently poor results, which usually leads to their final rejection. Errors in wave-lengths or intensities in Rowland's table, or close lines in the star, may be responsible for the trouble. However that may be, there is undoubtedly among the lines retained a large percentage which are in error similarly, but in a smaller degree. It simply remains to eliminate such lines as soon as they can be detected, though the uncertainty of proceeding in the dark in this regard is far from satisfactory.

There is a possibility of actual introduction of this form of error in the use of blends formed by weighting lines according to their intensities in Rowland's tables. The occurrence of enhanced lines is one great obstacle. But, further, in determining wave-lengths by blending, it must be remembered that the intensities of Rowland's lines were intended only for purposes of identification, and were not prepared with sufficient care to warrant more exacting applications. The characteristics as regards sharpness of lines comprising a blend would undoubtedly affect the apparent blended wave-length, though such characteristics could not be included in intensity estimates without considerable difficulty. Again, an intensity 0 applies to all lines between $a+0.5$ and $a-0.5$ in Rowland's scale; or, in other words, the uncertainty of any intensity in Rowland's table amounts to ± 0.5 of a unit. The possible error in a blend of this kind is well illustrated by the well-known combination of *Ti* (2), 4427.266 and *Fe* (5), 4427.482 into a line of wave-length 4427.420. If, however, the two intensities are 1.5 and 5.5, or again 2.5 and 4.5, the resulting wave-lengths are 4427.436 and 4427.405 respectively. Thus, under the assumption that the intensities in Rowland's table are as accurate as possible, an uncertainty of 0.015 tenth-meter (the equivalent of 1 km in velocity) must be recognized in the blend. It would therefore seem advisable with present methods to confine one's choice to the

seventy-five or eighty good single lines that are available in most high-dispersion instruments, rather than to include tempting groups whose wave-lengths are subject to uncertainties in addition to those incident to the laboratory determinations.

e) *Elimination of these errors.*—For errors due to change of film and inaccuracies in the measuring engine, including temperature-change during measures, adequate remedy has been proposed, but no elimination of the remaining uncertainties incident to present methods seems possible without a radical change in the system employed. As long as the wave-lengths determined from measures with the grating or interferometer by an observer of one personal equation, are employed by another observer of a different personal equation to represent his measures of corresponding lines as produced by a different spectroscope, it can hardly be expected that the results of various observers will be entirely consistent. The problem demands that the conditions which obtain in the production, measurement, and reduction of a spectrographic plate of a star should be exactly duplicated in the production, measurement, and in general the reduction, of the plate from which the fundamental data for velocity determinations are secured. The same spectrograph should produce both plates. They should be measured by the same observer with the same measuring engine. And, finally, they should be reduced in parallel. These requirements are clearly most exacting, but as a suggestion of a means toward this end I propose the following method, which, as far as I know, has never been applied in this way.

A PROPOSED METHOD FOR THE MEASUREMENT AND REDUCTION OF SPECTROGRAMS.

Proceeding as in case of a stellar exposure with the spectrograph in final adjustment from which it must not be disturbed, a source of accurately known velocity, such as the sky or Sun, is photographed with the comparison in the usual manner. Then the measures upon the comparison and continuous spectra of this plate with some engine by any observer will constitute his fundamental standard table for the engine and spectrograph used. These measures of the bright and dark lines of a plate of the sky or Sun and spark will fix the relative positions of the Fraunhofer and comparison lines for a known velocity, and will constitute a velocity-standard table. For the

determination of the velocity of any celestial object of the same characteristics as the standard source it is necessary only to duplicate the exposure and measures of the standard plate and compare directly the relative positions of corresponding lines of the two sources referred to the comparison spectrum of each plate. In the reduction of a stellar plate for velocity determination, we first reduce the star measures to the dispersion of the standard plate by forming a simple plot with micrometer readings as abscissæ and differences in settings on comparison lines as ordinates. For the star lines the reductions to fundamental dispersion are read directly from this plot. If these reductions are applied to the readings on the stellar spectrum, the measured positions of any line on the Sun and star plates will differ by an amount proportional to the relative velocity toward the Earth of the star and Sun.

The assumptions in this method are fundamental in every case of velocity determination. Further assumptions treated above as incident to other methods are here eliminated by the comparison of artificial lines with artificial lines, and dark lines with dark lines. The advantages of this simple system need only be mentioned. Errors in the screw of the measuring engine are eliminated for small displacements, such as are found in stellar spectra, if the standard and star plates occupy identical positions on the plate-carriage during measures. Personal equation is well controlled without the reversal of the spectrogram, and may be closely followed by frequent measures of standard plates. It may even seem preferable not to reverse the plate, for, as I have suggested, the curvature of lines may introduce a variation of this unaccountable source of error. The magnitude of accidental errors of setting is not affected by the use of this method of relative measures, but with the greater number of lines available in solar-type stars particularly, the reliability of the means from any plate is clearly increased. Uncertainties due to imperfect focal conditions are not only largely eliminated if the exposures on comparison and star are consistent in star and standard plates, but the region of spectrum available for measurement can be extended without fear of encountering difficulty. The number of lines available is further augmented for solar-type stars by the possibility of including blends, which are clearly reliable under this method. Until tests

are made with high-dispersion instruments it is difficult to predict the extent of the advantage of this increase of the measurable lines for this case; but for low-dispersion instruments, in my own experience with a magnifying power of 25, the number of available lines increased from 20 to 170; and with a power of 10, though practically no lines could be used with methods depending on wave-lengths, 65 lines gave good results for velocity determinations. But for the present the chief advantage of the method lies in its independence of the relative or absolute values of the wave-lengths or intensities of the spectral lines of the spark or continuous spectrum. For the determination of the factors necessary to convert micrometer displacements into kilometers, rough relative values of the wave-lengths of three favorable spark lines are needed; but, aside from this, there is no necessity for data of this kind. It is possible to make final definitive measures at once.

In the application of this method any degree of refinement can be introduced at the will of the observer. The number of plates employed to form a standard table can be increased until the full possibilities of the spectrograph and measuring engine are realized. By comparison of standard plates any errors due to temperature-change in the instrument can be detected, and tables can be prepared for different temperatures, if desired. Standard plates can be prepared occasionally and measured, along with the regular observing list, as checks on the constancy of the results. For absolute checks of the performance of the method, lunar and planetary spectrograms are of course available, but the determination of the sky or solar velocities furnish equally reliable checks, and at the same time afford data for strengthening the original standard tables. In general, three sky or Sun plates should furnish sufficient data for a fundamental table, though the conditions obtaining in any particular case might call for a greater or smaller number.

As a source of known velocity for the standard plates, the Sun is perhaps superior to the sky, but if exposures on the sky are more convenient, there should be little, if any, hesitation in their use. The objection to the use of sky-light lies in the fact that the lines in its spectrum are subject to a slight broadening due to the Sun's rotation. This would hardly exceed 0.020 tenth-meters,which would

occasion no impairment of the measures of lines which in most favorable cases are many times this amount in width. As star-light itself possesses the integrated characteristics of sky-light, it may seem preferable to use the sky in the preparation of standard plates.

This velocity-standard method is best applicable to solar-type stars, but it may, if desired, be modified to suit the requirements of hydrogen or other stars without the introduction of errors greater than those incident to present methods. Thus a standard table for any class of stars may be prepared by introducing in the spark, in addition to the regular comparison, those elements which produce good lines in the stellar spectra. The assumption is made that the relative positions of the spark and dark lines of the elements used are not affected differently either by the dissimilar physical conditions in the spark and star or by the peculiarities of the spectrograph; but this is an assumption commonly made in all spectrographic work.

II. AN APPLICATION OF THE VELOCITY-STANDARD METHOD.

(*a*) *Apparatus employed.*—Necessity for some departure from the present methods of measurement and reduction of spectrograms first arose in the attempt to improve the accuracy of relative velocity determinations from spectrum plates made with Spectrograph I of the Lick Observatory. The performance of this instrument is well known in connection with the spectrographic studies by Campbell, Wright, H. D. Curtis, and Stebbins, of the spectra of *Nova Persei*, *Nova Geminorum*, and *o Ceti*, and with velocity determinations by Campbell of *1830 Groombridge*. As the result of this work, Professor Campbell was led to remark that

> The greatest interest in the observation lies in the fact that fairly accurate determinations of stellar velocities are shown to be possible down to the eighth or ninth photographic magnitudes, provided the spectra contain well defined lines.

The importance of permanent adjustment of this spectrograph was realized from the first, but the use of nearly all its parts in the Mills spectrograph necessitated a practical reconstruction of the single prism instrument whenever it was used. The Mills spectrograph was recently rebuilt, and all the parts of Spectrograph I were made permanently available, with the exception of the collimator lens, the slit-head, and the comparison-spectrum apparatus. Professor Camp-

bell asked me to design these parts in order to complete the instrument.[1] A slit-head[2] was adapted from Keeler's star spectroscope, and the comparison apparatus was patterned closely after the device proposed by Wright.[3] The new collimator lens was essentially a counterpart of the old one. The focal plane of the camera lens was found to be nearly flat. No wedge was required under the plate-holder with $H\gamma$ light at minimum deviation, and excellent definition for velocity work was secured from λ 3800 to λ 4600.

A brief description of Spectrograph I appears in ***Bulletin*** No. 8 of the Lick Observatory. A more detailed description of some of the parts is given in Director Campbell's article on the Mills spectrograph; but for the sake of completeness the principal constants will be repeated here.

Focal length of collimator - - - - - -	720 mm
Aperture of collimator - - - - - - -	37 mm
Refracting angle of prism - - - - - -	60°
Focal length of camera lens - - - - - -	406 mm
Dispersion per t.-m. at $H\gamma$, - - - - -	8″

The performance of this spectrograph is exceptionally good. The limit of resolving power with lantern-slide plates is about 0.50 tenth-meters at $H\gamma$, which is all that could be expected of an instrument of one-fifth the dispersion of the Mills. However, with rapid emulsions, the increased size of silver grains interferes seriously with the definition. Careful tests by Dr. J. Stebbins and Dr. J. H. Moore have shown that all effects of flexure have been successfully eliminated in the construction of this instrument. But the effect of temperature-change on the position of lines is very marked. This is largely due to the unequal expansion in the triangle formed by the steel collimator tube, the brass camera tube, and the brass tie-rods which extend from the collimator tube nearly to the plate-holder. The extent of this shift has been determined by Dr. Moore. It is equivalent to approximately 36 kilometers in velocity per degree Centigrade change

[1] All these improvements, as well as a constant-temperature case and thermostat, have been supplied by a grant from the Draper Fund of the National Academy of Sciences.—W. W. C.

[2] Described in *Publications of the Lick Observatory*, **3**, 174.

[3] ASTROPHYSICAL JOURNAL, **12**, 274, 1900.

in temperature. The limitations of the instrument are thus readily appreciated. In my own work, in the absence of a temperature case, the spectrograph has been wrapped in several thicknesses of woolen blanket just before the exposure began. At the same time all windows in the dome were closed. These precautions, together with the frequent introduction of the comparison spectrum, have made good results possible, but it has been almost invariably the case that the best velocity determinations have been made from plates for which the temperature range was a minimum. Recently a temperature case and automatic thermostat have been constructed for this instrument. These may be expected to result in very material improvement in its performance.

Further disadvantages are inherent in an instrument of low dispersion and low resolving power. Linear defects in lines produce five times the error that they would occasion with the Mills spectrograph. Probably no more than fifteen or twenty single lines could be measured with any power on any one plate of a solar-type star. Stebbins found only six coincidences of solar and stellar lines for *o Ceti* in a region of 340 tenth-meters, while in a region covering 120 tenth-meters he found twenty such lines on Mills plates. With a power of ten, absolutely no single lines would be available for velocity determinations. In view of these facts, it would seem that few tests could be more exacting than the application of the velocity-standard method to this case. Spectrograms made with an instrument of this kind with no greater protection against temperature-change are affected by errors which accurate measurements serve to bring out; but it was one aim of the author to determine the degree of accuracy to be expected from an instrument of low dispersion, for its field of usefulness is practically inexhaustible, as it reduces the exposure time required with the Mills spectrograph by 90 per cent.

(*b*) *The fundamental velocity tables.*—Following the plan of the method above described, Zero-Standard Tables II and III have been prepared from three sky and iron plates with a Toepfer measuring engine, whose least reading is 0.00025 mm. These tables will be directly useful only to myself in the reduction of plates made last year, but will serve as a complete illustration of the method, if not as

guides to other observers in the choice of lines. Corrections for radial velocity and diurnal rotation of the Earth have not been applied, as they were considered negligible. The fundamental dispersion was obtained from a star plate at the mean temperature for the series to be measured. To this dispersion all the plates have been separately reduced. The results for all lines of wave-length greater than λ 4400 are inferior, for the reason that beyond this point the sensitiveness of the lantern-slide emulsion drops suddenly, and the character of the iron lines suffers marked decline; but it has been considered wise to include them in the measures, as the three comparison lines at λ 4900 afford a good control for the curve.

Wave-lengths of all the lines sufficiently accurate for purposes of identification appear in the first column of both tables. The second column contains the factors to convert micrometer displacements into kilometers. The quantity, V_s, which enters into these factors, was taken directly from Frost's "Scheiner's Astronomical Spectroscopy."

Columns 3 and 4 of Table II contain the micrometer readings on the star and iron lines respectively. The number of measures on which each reading depends is given in column 6. In addition to the three standard plates, five other star plates were included in the determination of the iron lines.

All measures for Table II were made with a magnifying power of twenty-five, while Table III was prepared for a power of ten or twelve. In forming the original low-power tables, the measures of two standard plates with a power of ten were separately reduced to the fundamental dispersion by comparison with the high-power table of comparison lines, which could then be adopted for the comparison-line readings of the low-power tables. On comparison of this table with measures of star plates some discrepancies were found. These would arise from actual errors in the table or a difference in the wave-lengths of the lines in Sun and star. Several measures in Table II which came into agreement with the low-power star measures were then introduced into the low-power table. Seventeen of the best of the remaining new or discordant lines in the star were determined from thirty plates and are included in Table IV.

In the measurement of thirty plates of *W Sagittarii* made in 1903, the one aim was to obtain the best possible relative velocities without any sacrifice of absolute determinations. As the mean error in the setting of any line in the original table was found to be equivalent to about 2.3 km displacement, it was conceivable that a velocity determination from the lines measurable on any one plate would differ slightly from a corresponding determination from any other plate, due to errors in the fundamental table. To eliminate this possibility a rather laborious method of reduction was employed. In the first place, all the plates were reduced by the original table. The residual for each line was then formed for every plate, and the mean of all the residuals for any line was applied as a correction to the reading for that line in the original table. The weighted mean of the readings for any line occurring in the original table and in the table thus prepared from thirty plates was adopted as the final setting for that line. These means for all the lines appear in column 3 of Table III. The relative accuracy of these values thus depends upon measures of thirty-three plates, while their absolute accuracy depends upon the three original standard plates. With this new table, all the plates were reduced again.

As a further move toward greater relative accuracy, the seventeen additional star lines which were determined from thirty plates were used with the original standard lines in the reductions. In Table IV the results for these seventeen lines are given under headings similar to those in Table III.

The settings for the iron lines in column 4 of Table III depend upon low-power measures of thirty-one plates which were reduced to the fundamental dispersion, and averaged. Column 5 contains the probable errors of the quantities appearing in the two preceding columns as determined from the observations on star plates only. The number of these observations is given in column 6.

In column 5 of Table II and column 7 of Table III a brief description of each line is given. The character of the line is indicated by the letters vG (very good), G (good), F (fair), and P (poor). Then follows the intensity according to Rowland's scale. The remaining comments are self-explanatory, but include the following arbitrary

abbreviations: *b*, broad; L, line; Gr, green; V, violet; inc, included; shp, sharp; gp, group; B, bright; Cl, close; I, intensity; and F, faint.

Column 8 of Table III contains the differences between the settings for the various lines in the original solar table and the corresponding values in the final table in column 3. Regarding these as residuals for the measures of the original table, the probable error of the absolute velocities as affected by the errors in the original low-power table is seen to be ± 0.24 km.

No correction for curvature has been introduced into either of these tables. It should be noticed that this correction is entirely differential in its nature with this method of measurement and is negligible if the slit-length is not altered. The formulæ for the computation of this small correction, based on the assumption of a parabolic curvature of the spectral lines, are as follows:

$$\text{For Table II,}\quad dV = -(y^2-1.2)\ 0.5\ \text{km},$$
$$\text{For Table III,}\quad dV = -(y^2-1.1)\ 0.46\ \text{km},$$

where y is the distance in units of $\frac{1}{4}$ millimeter from the center of the spectrum to the measured part of the comparison line.

TABLE II.

Standard Table (Magnifying Power 25).

Wave-Lengths	rV_s	Micrometer Readings		Notes	Observations
		Sky	Iron		
3969.4			13.879	G λ3969.413	7
3982.0	760	14.928		F−G10*b*	3
3987.0	767	15.634		G 20	3
3994.7			16.668	F	3
4005.3			17.421	G 4005.370	8
4005.3	778	17.423		G 20	3
4006.8	780	17.562		F 15*b*	3
4012.5	781	18.100		F 10*b*. Shp. Ls to V	3
4013.9	782	18.252		F 10*b*	3
4014.2	782	18.288		F 20 Mean.	3
4014.6	783	18.338		P 8	3
4018.0	784	18.625		F 18	3
4021.8			19.009	F	2
4022.0	787	19.023		G 10*b*. LF6 at −.070	3
4025.0	789	19.301		F 15. f Ls to V.	3
4028.6	791	19.634		F−P 8. *b*.	2

TABLE II—*Continued.*

Wave-Lengths	rV_s	Micrometer Readings		Notes	Obser-vations
		Sky	Iron		
4029.8	792	19.748		F 6	2
4030.9	792	19.845		G 15	3
4030.9			19.847	F−G	8
4032.0	793	19.978		P 8	2
4033.0	793	20.057		F−G 10*b*	3
4034.6	794	20.201		G 9	3
4035.8	795	20.313		G 8	2
4040.2	797	20.711		F 5	2
4040.9	798	20.778		F 7	2
4041.7	800	20.853		F 8*b*	3
4044.0	801	21.083		G 8	3
4044.8	801	21.162		F 6	3
4046.0			21.249	G 4045.975	8
4046.0	802	21.252		G 20	3
4048.8	804	21.550		F 7	3
4052.4	806	21.871		F 18*b*	2
4055.3	807	22.090		F 18*b*	3
4057.6	809	22.330		G 10	3
4063.7			22.878	G 4063.705	8
4063.9	812	22.882		G 17. L at .075 not inc.	3
4071.9			23.613	G 4071.895	8
4072.0	818	23.615		G 15. L at .075	3
4077.9	822	24.174		G 10	3
4078.6	822	24.252		P 6*b*	3
4079.4	823	24.313		F 8*b*. F Ls to Gr	3
4092.7	832	25.463		G 12	3
4096.2	836	25.787		F 8	3
4098.5	836	25.978		F 10	3
4100.2	838	26.115		F 7. Mean 2 cl. Ls.	3
4104.4	840	26.524		F 6. LF 8*b* at +.060	3
4106.5	842	26.668		F 5.	3
4107.6			26.777	F	4
4107.7	842	26.782		F 7	2
4115.0	847	27.418		F 12*b*	3
4116.7	848	27.561		F 10*b*	2
4118.8			27.730	G 4118.806	8
4118.9	850	27.741		G 12	3
4121.8	851	27.997		F−P 10 Mean 2 Ls	3
4123.9	853	28.163		F 8*b*	3
4126.0	854	28.339		G 7	3
4126.2	854	28.360		G 15*b*. Mean	3
4126.5	855	28.392		G 7	3
4128.0	856	28.518		F 12*b*.	3
4132.5			28.881	G	8
4132.6	859	28.892		G 15	3
4134.0	859	29.043		F 8	3
4134.5	860	29.077		G 18	3
4134.7	860	29.101		F 10*b*	3
4143.9	867	29.865		G 15	3
4144.0			29.866	G 4143.955	7
4150.0	871	30.358		P 20	3

TABLE II—*Continued.*

WAVE-LENGTHS	rV_s	MICROMETER READINGS Sky	MICROMETER READINGS Iron	NOTES	OBSERVATIONS
4152.3	872	30.553		G 10	3
4154.1	873	30.715		F 7	3
4154.4	873	30.741		G 15*b* Mean.	3
4154.6			30.739	F 4154.571	6
4154.8	874	30.771		F 7	3
4156.5	874	30.893		F 8	3
4156.8	875	30.947		G 15	3
4157.0	875	30.923		G 8	3
4157.9	876	31.044		P 8	2
4160.7	878	31.275		F 8*b*	3
4161.2	878	31.310		G 12	3
4161.5	878	31.336		F 8*b*	3
4163.8	879	31.539		F 8	3
4165.7	880	31.654		F 8*b*	3
4167.4	882	31.819		vG 15	3
4171.2	884	32.118		F 7. Shp.	2
4172.9	886	32.264		F 7	3
4175.1	887	32.438		F 6. Shp.	3
4177.8	888	32.652		F 15. Mean.	3
4182.2			32.985	G. 4182.155	8
4182.3	891	32.988		G 10. L.F5 at +.067	3
4187.3	894	33.414		G 8	3
4187.6	894	33.433		G 17. Inc. L 6.4186.9	3
4187.6			33.438	F–G 4187.572	8
4187.6	894	33.438		G 15. Mean 4187.3 +4187.9	2
4187.9	895	33.465		G 8	3
4188.9	896	33.562		G 5	3
4191.7			33.746	F 4191.678	6
4191.6	897	33.766		G 10*b* L.P6+.080 not inc.	3
4198.5	901	34.317		G 10	3
4198.6	901	34.326		G 16 4198.5+4199.2	3
4198.7			34.351	G	8
4202.2			34.594	G 4202.185	8
4202.2	903	34.603		G 10 L.F7 at −0.080	3
4207.0	906	34.992		F 9*b*.	3
4216.0	912	35.676		G 20	3
4219.5			35.955	F	8
4219.5	914	35.956		F 8	3
4222.3			36.174	F	5
4222.3	916	36.175		P 7. cl.Ls.	2
4226.9	919	36.524		F 15 "g"	3
4227.6			36.578	G	8
4227.6	919	36.580		F 9	3
4229.8	920	36.757		G 10	3
4233.5	923	37.031		G 15	3
4233.6			37.040	G–F	8
4236.1	925	37.228		G 12	3
4236.1			37.230	G. 4236.11	8
4243.0	930	37.765		G 30. Shp. edges	3
4250.4	936	38.307		G 8	3

TABLE II—*Continued.*

Wave-Lengths	rV_s	Micrometer Readings		Notes	Observations
		Sky	Iron		
4250.8	936	38.337		G 17 Mean.	3
4250.6			38.343	G 4250.62	8
4251.1	936	38.368		G 8	2
4254.4	939	38.635		G 10	3
4260.5			39.083	G 4260.50	8
4260.4	942	39.084		G 12.	3
4271.7	950	39.910		G 18	3
4271.8			39.916	G 4271.76	8
4275.2	952	40.152		F 25, 0.120−0.180	2
4280.5	957	40.541		F 15. Close Gp.	3
4282.6			40.700	G. 4282.56	8
4283.0	959	40.709		G 10*b*	3
4289.7	962	41.229		F 20, 0.180−0.270	3
4294.3			41.555	G 4294.30	8
4294.2	967	41.567		F 8	3
4297.0	968	41.765		G 12. 0.730−0.800	3
4299.4			41.924	G 4299.41	8
4299.6	969	41.939		F 20*b*	3
4301.7	970	42.035		F 7	2
4308.1			42.546	G 4308.08	8
4308.1	976	42.553		F 15	3
4314.1	980	43.002		F−P 7	3
4314.5	980	43.033		G 20. Gp. 0.970 −1.090	3
					3
4315.2	981	43.062		F−P 7	3
4318.8	983	43.301		G 10	2
4321.0	984	43.440		F 10	2
4321.9	985	43.497		F 5	3
4323.6	987	43.654		G 20	3
4325.3	988	43.776		F 80	3
4325.8	989	43.808	43.811	G 20	3
4326.4			43.811	G 4326.40	8
4326.6	990	43.828		F 8	3
4331.2	992	44.159		F 10*b* Gp.	3
4337.2	997	44.608		F 9*b*	3
4337.6	998	44.634		F 18	3
4338.0	998	44.657		F 8	3
4340.7	999	44.835		G 20 "$H\gamma$."	3
4344.1	1002	45.093		F 12*b*	3
4346.7	1003	45.254		G 8	3
4352.0	1007	45.611		G 8	3
4352.3	1007	45.641		G 18 0.600−0.685	3
4367.8	1019	46.711		F 10 Shp.	3
4370.0	1021	46.844		F−P 8	3
4371.2	1022	46.930		F 8	2
4374.7	1024	47.144		F 8*b*	3
4375.4	1024	47.189		F 18	3
4376.1	1025	47.230		F 8	2
4383.7	1031	47.758		G 25	3
4383.7			47.764	G 4383.67	8
4385.2	1031	47.866		F 9	3
4399.9	1041	48.844		F 6	3

TABLE II—*Continued.*

Wave-Lengths	rV_s	Micrometer Readings		Notes	Obser-vations
		Sky	Iron		
4400.6	1042	48.887		F 10*b*	3
4408.8	1042	48.904		G 25	3
4401.5	1043	48.939		G 8	3
4403.3	1044	49.060		G−F 7	3
4404.9	1045	49.163		G 12	3
4404.9			49.164	G 4404.93	8
4415.3			49.836	G−F 4415.29	8
4415.4	1052	49.841		G 15	3
4422.8	1059	50.303		F 10*b* 0.245−0.320	3
4427.5	1062	50.620		G 8*b*	3
4430.8	1064	50.823		F−G 15	3
4435.5	1068	51.140		F−P 20	3
4447.5	1077	51.908		G 15.	3
4451.2	1079	52.093		F 12	3
4453.3	1081	52.267		F 5	3
4455.1	1082	52.381		G 12	3
4458.4			52.650	G−F	7
4462.0	1088	52.808		G 12	3
4466.7	1092	53.105		F 7*b*	2
4466.7			53.106	F−G 4466.73	8
4476.2			53.688	G 4476.18	8
4482.4	1102	54.060		F 8	2
4482.4			54.063	F−G 4482.39	8
4494.7			54.814	G 4494.74	8
4494.8	1112	54.815		F 10*b*	3
4501.4	1118	55.230		F 20	3
4518.0	1130	56.209		F 8*b*	3
4520.4	1132	56.342		F 7	3
4522.9	1134	56.495		G 8	3
4525.3	1136	56.642		F 6	2
4526.8	1137	56.735		F 8	3
4528.9			56.846	F−G	8
4529.0	1138	56.852		F 8*b*	3
4531.3	1140	56.987		G 9	3
4545.0	1151	57.795		F 8*b*	3
4549.8			58.046	F	8
4549.8	1153	58.048		G 10*b*	3
4552.8	1155	58.228		F 8	2
4554.3	1156	58.317		F 8*b*	2
4556.2	1158	58.436		G 12*b*	3
4563.9	1162	58.866		F 6	3
4571.8	1169	59.333		F 15	3
4584.0			60.009	F−G. 4584.02	8
4590.2	1183	60.340		F 8	3
4598.1	1188	60.778		F 8*b*	3
4611.3	1200	61.656		G 7	3
4630.—			62.580	F	8
4871.9			73.896	G 4871.88	5
4859.9			74.430	F 48.5993	2
4891.4			75.301	G 4891.37	5
4920.7			76.560	F 4920.68	1
4957.7			78.156	G 4957.67	5

TABLE III.

Standard Table (Magnifying Power 10–15).

Wave-Lengths	rV_s	Micrometer: Sky and Star	Micrometer: Iron	P. E.	Observations	Remarks	Reduction from Sky Plates
3969.4			13.8790	±0.0003	..		+0.001
4005.3			17.4189	0.0002	..	G	+0.003
4005.3	780	17.424*		0.0008	26	G 25. Close BL to Gr.	+0.001
4012.5	780	18.104*		0.0010	25	F 13. 0.080–0.150. Cl Ls to V.	+0.004
4024.8	790	19.289		0.0016	19	G 30*b*	+0.003
4041.0	800	20.795		0.0014	24	F 25. 0.740–0.840=3Ls.	+0.002
4046.0			21.2472	0.0003	..		
4045.9	800	21.252*		0.0009	27	G 20. Cl. Ls to V.	−0.005
4063.7			22.8785	0.0003	..		
4063.9	810	22.881*		0.0009	20	F–G17.	−0.001
4066.8	810	23.187		0.0012	18	G15.	+0.002
4067.2	810	23.211		0.0010	18	F 30 Mean.	+0.001
4072.0	820	23.612*		0.0009	22	G 20.	−0.007
4071.9			23.6127	0.0003	27		
4092.8	830	25.462*		0.0009	14	G 12. L F12 at −0.160.	−0.001
4101.7	840	26.270		0.0011	25	G 30. 0.220–0.350. *Hδ*.	0.000
4118.8			27.733		2		
4118.9	850	27.747*		0.0009	23	G12 L+0.100 F6.	−0.004
4128.0	860	28.523		0.0015	16	F12*b*. Ls F15 at −0.150 and at +0.144	+0.002
4132.5			28.8831	0.0003	28		
4134.5	860	29.086		0.0016	17	G20	+0.001
4137.0	860	29.315		0.0013	20	G15. In *b* region.	−0.003
4143.6	870	29.840		0.0010	9	G25. Weight I. L.F12 at −0.142 not inc.	−0.001
4143.9	870	29.863		0.0013	25	G 12	−0.003
4144.0			29.8655	0.0002	28		
4150.0	870	30.364*		0.0011	25	G–F20	−0.002
4152.3	870	30.556		0.0013	18	G 10	0.000
4154.4	870	30.741*		0.0009	28	G 15	+0.002
4156.8	870	30.923*		0.0008	28	G 15	−0.002
4061.2	880	31.319*		0.0009	28	G 12	+0.005
4163.6	880	31.521*		0.0008	20	G 11	0.000
4178.5	890	32.712*		0.0008	28	G 40. L+0.130 not inc.	+0.004
4182.2			32.985		8		
4182.3	890	32.995*		0.0009	22	G 10	0.000
4184.7	890	33.178		0.0016	17	F 20. Mean of 2Ls.	+0.003
4187.6	890	33.434*		0.0010	27	G 17. LG 5+0.125 not inc.	−0.003
4191.7	900	33.771		0.0018	12	G–F10.	−0.001
4196.0	900	34.109		0.0013	26	G–F25. Mean of 2Ls.	+0.004
4198.5	900	34.325*		0.0009	22	G 20.	+0.004
4198.6	900	34.339		0.0013	5	G 25. Inc.L.5 4199.3	−0.001
4198.7			34.3524	0.0003	28		
4202.2			34.5927	0.0004	28		
4202.2	900	34.603		0.0011	22	F 25	+0.001
4216.0	910	35.673*		0.0009	28	G 25. L 4217.7 A.U. not inc.	0.000
4227.0	920	36.539		0.0011	23	F–G30.	+0.001
4227.6			36.584		6		
4233.5	920	37.028*		0.0008	28	G–F15.	−0.005
4233.6			37.042		8		
4236.1			37.233		9		
4236.2	920	37.234		0.0011	16	F20. Whole L. Weight 1.	−0.004
4236.3	930	37.252		0.0014	9	Middle of L.	0.000
4250.6	940	38.341*		0.0013	25	G 17	0.000
4250.6			38.3417	0.0003	27		
4260.5			39.0834	0.0003	28		
4260.4	940	39.089*		0.0011	25	G 12. LF10 at −0.140	+0.001
4271.7	950	39.911		0.0014	29	G 20	−0.003
4271.8			39.9139	0.0003	29		
4275.1	950	40.154*		0.0007	18	F–G25 0.120–0.180	0.000
4282.6			40.6989	0.0003	20		
4283.2	960	40.721*		0.0011	24	F10*b*.	0.000
4290.0	960	41.231		0.0011	6	F20 0.180–0.270	+0.002
4294.3			41.5562	0.0004	28		
4294.3	970	41.562		0.0015	13	F10	−0.005
4294.4	970	41.580		0.0022	12	G20. Cl.L to Gr	−0.002
4299.4			41.9266	0.0004	25		

TABLE III—*Continued.*

Wave-Lengths	rV_s	Micrometer		P. E.	Observations	Remarks	Reduction from Sky Plates
		Sky and Star	Iron				
4300.4	970	41.964		0.0011	28	G30 LP18 to Gr not inc. Wide group.	−0.003
4306.2	970	42.382		0.0013	27	F−G10	+0.004
4308.1			42.5472	0.0003	28		
4326.4	990	43.806*		0.0010	27	G20	−0.006
4326.4			43.8096	0.0004	29		
4331.2	990	44.154*		0.0010	27	F−G10	+0.001
4340.7	1000	44.839*		0.0009	27	G20 *Hγ*	+0.003
4344.1	1000	45.095*		0.0010	29	F15*b*	+0.002
4375.4	1020	47.186*		0.0008	30	G20	0.000
4383.7			47.7651	0.0003	29		
4395.0	1040	48.544*		0.0011	27	G−F15 Ls to V.	−0.004
4401.2	1040	48.895		0.0011	16	Dense part to Gr.	−0.004
4404.9	1050	49.161		0.0013	24	G12 LP8 at −0.070	+0.003
4404.9			49.1636	0.0002	29		
4415.3			49.8347	0.0003	27		
4415.3	1050	49.837*		0.0010	28	GF15	+0.002
4422.8	1060	50.308		0.0012	18	F15*b* 0.246−0.360. LF10+0.100.	+0.001
4443.0	1070	51.647*		0.0010	28	F25 0.590−0.720.	0.000
4451.3	1080	52.102*		0.0011	21	G18. G12 in V. Measure whole line.	−0.001
4455.4	1080	52.397		0.0014	22	F25 Mean of 2Ls.	+0.003
4462.0	1090	52.806		0.0014	24	F12	+0.001
4466.7			53.1055		10		
4476.2			53.6887	0.0007	21		
4482.0	1100	54.002		0.0015	22	F25	−0.002
4494.7	1110	54.806		0.0024	12	F10*b*	+0.002
4494.7			54.8121	0.0007	25		
4501.3	1120	55.220		0.0011	22	F20. Measure sharp middle.	−0.005
4528.9			56.8404	0.0014	7		
4541.0	1150	57.552		0.0017	9	F15*b*. Ls to V not inc.	−0.002
4549.8	1150	58.056*		0.0010	23	G20	+0.003
4571.8	1170	59.332*		0.0009	24	F15	+0.005
4584.0			60.0115	±0.0008	23		
4630			62.5809		16		
4871.9			74.4299		16		
4891.4			75.3012		17		
4957.7			78.1556		17		

TABLE IV.

Additional Lines in Star (Low Power).

Wave-Lengths	rV_s	R	P. E.	Obs.	Remarks
4063.6	810	22.860	±0.0011	8	F25 Wt. I.
4132.7	860	28.905	0.0012	27	G15
4137.3	860	29.338	0.0009	18	25 whole line
4191.4	900	33.751	0.0013	20	GF20. L to V. included.
4204.7	900	34.834	0.0016	20	F18. LF12 at +0.120.
4290.1	960	41.257	0.0010	27	F30 Weight I. LP12 at −0.140.
4297.0	970	41.745	0.0015	11	F−G12 0.700−0.800
4314.4	980	43.026*	0.0009	25	G20 0.970−1.090
4334.7	990	44.372	0.0014	17	F−G18. L at +0.100 not inc.
4337.8	1000	44.648*	0.0010	27	F−G25
4352.0	1010	45.611*	0.0011	29	G30 L at 0.150 not inc.
4360.0	1010	46.113	0.0011	24	F−G20
4384.5	1030	47.814*	0.0009	29	G38
4401.0	1040	48.883=	0.0006	19	G25 L at −0.170 not inc.
4417.9	1050	50.002	0.0014	17	F20 Mean of 2Ls.
4466.4	1090	52.974	0.0017	12	F10
4583.7	1180	59.988	±0.0011	17	F−G20

(*c*) *Application to the study of the variable star, W Sagittarii.*—Exhaustive studies of the light curve of *W Sagittarii* were made by J. H. F. Schmidt, who discovered the variability of this star, then known as *γ′ Sagittarii*, at Athens in 1866.[1] From a series of 890 observations covering ten years and extending over 195 maxima and 193 minima he constructed a light-curve, which is drawn in detail in Fig. 1 of this paper. In addition to the strong irregularities in the curve, he suspected a perturbation in the light-period running through a cycle of eight years, affecting the time of maximum and minimum by several tenths of a day. But his observations were hardly extensive enough to warrant this last conclusion. The spectrographic observations at present available do not cover a period long enough to confirm or to disprove this result. As presented in the Chandler and Harvard Catalogues, the important data regarding this star, with some additions, are as follows:

CHANDLER'S THIRD CATALOGUE.

Max., $4^{m}.8$; Min., $5^{m}.8$; M–m, $3^{d}.00$; Period, $7^{d}.59460$ E.
Epoch of Max., 1866 Sept. 4; Julian, $2402849^{d}.45$.

HARVARD CATALOGUES.

Max., $4^{m}.3$; Min., $5^{m}.1$; Class IV; Sp., G5K.
α, 1900.0, $17^{h}\ 58^{m}.6$; δ, 1900.0, $-29°\ 35'$.

ADDITIONAL DATA.

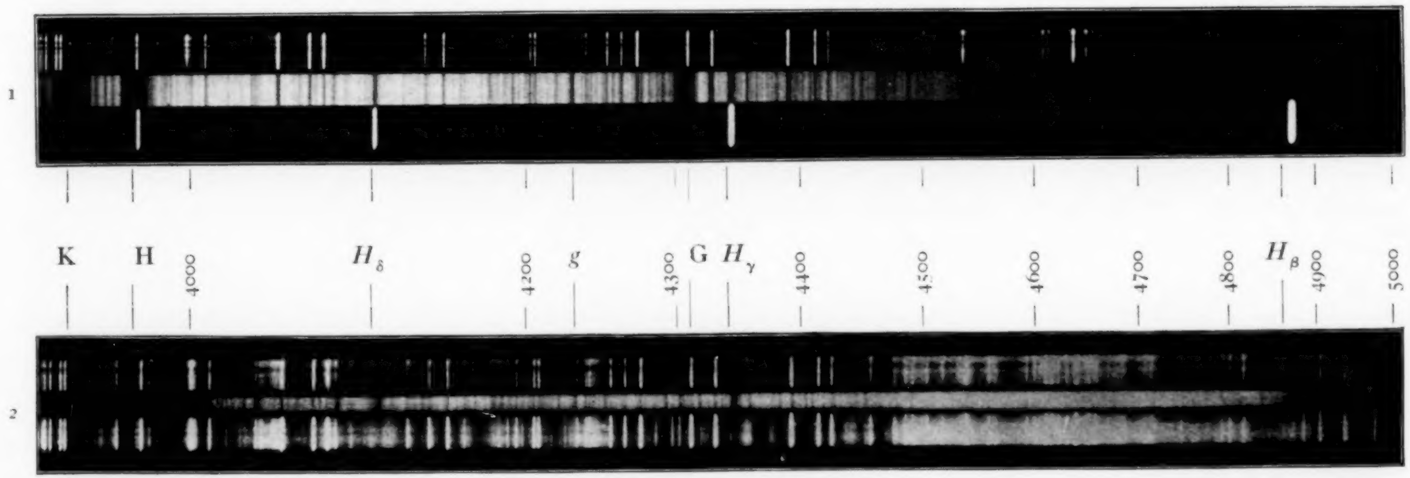
λ, 1902.0, $269°\ 44'$; β, 1902.0, $-6°\ 8'$
Photo. Max., $5^{m}.5$; Photo. Min., $6^{m}.5$.

The spectrum *W Sagittarii* approximates very closely to the solar type. The fact that sixty-five blends ranging in intensity from 12 to 40 gave good results, as exhibited in the last column of Table III, establishes the close resemblance of this star to our own central body. The seventeen lines in Table IV suggest some differences, the most striking of which is the strong line at λ 4334.7, which is not found in the Sun, but rises to an intensity of 12 in the star. But, on the whole, the identities are far-reaching enough to warrant an extensive system of blending.

This fact justified the use of a power of ten in the measures of the

[1] *Astronomische Nachrichten*, **87**, 103, 1876.

PLATE XIII.



1. Spectra of $\left\{ \begin{array}{l} \text{Iron.} \\ \text{Sky.} \\ \text{Hydrogen.} \end{array} \right.$

2. Spectrum of *W Sagitarii*, 1903, September 16. 77A.

plates of *W Sagittarii.* But the selection of such a low power was not decided upon without considerable experiment with a higher one. With power 10, the width of the micrometer thread (about 0.50 tenth-meters) interfered with settings on lines of intensity 12 to 15, while the density of the comparison lines was so great that it became difficult at times to distinguish the black vertical wire when set on them. After experimenting with powers of 10, 12, 15, and 25, I am inclined to think that a power in the neighborhood of 15 would perform most satisfactorily in the case of solar-type stars.

From preliminary measures in August 1903 of three spectrograms of *W Sagittarii*, a very considerable variation in the radial velocity of this star was brought to light. Subsequently a series of thirty-three spectrograms, distributed uniformly in the light-period, were secured with Spectrograph I. These have been measured and reduced for the purpose of determining the character of the orbit of this interesting variable. I have not been able to recognize upon these plates any effect attributable to the light of the fainter companion now known to exist, but it should be remembered that a difference of one magnitude is nearly sufficient to obliterate the effect of the darker star.

The details of the production of these plates are recorded in the *Journal of Observations*, Table V. The longer exposures of the comparison were made with a diaphragm occulting all lines to the violet of λ 4415, while during the shorter exposures the entire plate was uncovered. When the comparison was introduced four times, it was photographed once with the diaphragm and once without on each side. When introduced six times, four exposures were made without and two with the occulting device. With ten introductions of the comparison, three exposures were made on each side with the diaphragm removed, and two on each side with the diaphragm in position. The slit-width is expressed in terms of the divisions of the slit micrometer-head. The unit is 0.025 mm. To illustrate better the methods employed, complete measures and reductions of Plate 56A are published in Table VII. The columns from left to right contain the wave-lengths of the spectral lines, the micrometer readings on the star, those on the comparison, the reduction to fundamental dispersion, the displacements of the star lines from the solar lines of the

zero-standard table, and the equivalents of these displacements in kilometers per second. Referring to the description of the plates in the last two columns in Table VI, and to the temperature-range in Table V, it is evident that 56A is about an average plate.

TABLE V.

Journal of Observations (*W Sagittarii*).

Plate	Astronomical Date	Mt. Hamilton Sidereal Time, Mid-Exposure	Duration	Comparison		Observing Room Temperature			Slit-Width	Seeing	Remarks
				Times	Length of Exposure	Begin C.	End C.	Range C.			
	1903	h m	m		s s	°	°	°	in.		
20B	Aug. 3	17 47	45	4	5 &25	18.3	18.2	0.1	1.4	Fair	
25A	9	17 40	40	4	4 &20–30	24.5	24.2	0.3	1.3	Good	
36D	Aug. 17	17 52	45	4	5 &25–30	20.8	20.7	0.1	1.4+	Poor	
46A	Sept. 2	18 21	30	6	3 &25	24.8	24.4	0.4	1.3	Good	
47B	4	18 38±3^m	35±5^m	6	3 &27	....	12.8	....	1.4	Very Poor	Wind 30 miles.
56A	7	18 9	45	6	3 &26	17.2	16.4	0.8	1.3+	Fair	
57B	7	18 49	28	6	3 &26	16.4	16.3	0.1	1.3+	Fair	
58A	8	18 9	25	6	3 &26	19.7	19.2	0.5	1.4	Fair	
59D	8	18 38	30	6	3 &26	19.2	18.8	0.4	1.4	Good	
60A	9	18 9	15	6	2.5 &26	21.7	21.4	0.3	1.3	Good	
61D	9	18 32	24	6	2.5 &26	21.4	....	0.3	1.3	Fair	
63A	10	18 7	20	6	2.5 &26	21.4	....	0.5	1.3	Fair	
64B	10	18 28	19	6	2.5 &26	....	20.5	0.4	1.3	Fair	
65A	11	18 4	15	6	3 &26	16.0	....	0.4	1.3	Fair	
66B	11	18 23	20	6	3 &26	....	15.3	0.3	1.3	Fair	
75A	14	18 33	35	6	2–3 &25	13.6	12.8	0.8	1.3	Fair	
76B	14	19 1	21	6	2–3 &20	12.8	12.6	0.2	1.3	Good	
77A	16	18 25	24	6	25	19.2	18.5	0.7	1.3+	Good	
78B	16	18 54	32	6	2–3 &20	18.5	18.0	0.5	1.3+	Good	
82A	20	19 1	83	6	2–3 &25	18.9	17.5	1.4	1.3+	Good	Clouds cut off ½ light.
83A	21	19 15	85	6	3–4 &23	18.4	17.8	0.6	.....	Poor.	Mid-comparison at 19h.0m. Smoky sky cuts off ½ light.
86A	23	19 11	65	6	3 &25	19.4	19.3	0.1	1.3+	Bad	
91A	24	18 45	30	6	3 &25	22.5	21.8	0.7	1.3+	Fair	
92B	25	18 56	43	6	3 &25	19.1	18.7	0.4	1.3+	Good	Smoke reduces image ½
93B	26	18 51	35	6	....	19.8	19.8	0.0	1.3	Good	Short circuit in comparison. Gave extra time.
94D	27	19 3	53	6	3 &25	16.7	16.6	0.1	1.3+	Poor.	
95B	Oct. 3	19 24	25	6	3–6 &25–40	11.0	10.4	0.6	1.3+	Fair	
99F	4	19 46	76	6	3–4 &26	11.7	11.0	0.7	1.3	Good to Bad	Image faint. Objective probably fogged.
100A	5	19 47	45	6	3.5–4 &25	11.8	11.3	0.5	1.3	Poor to Fair	
102A	6	19 36	39	6	3–4 &25–35	14.6	14.2	0.4	1.3	Good	
103B	6	20 13	33	6	3–4 &25–35	14.2	13.8	0.4	1.3	Good to Poor	
	1904										
136F	June 7	17 16	52	10	1.5 &11	14.6	14.0	0.6	1.3+	Poor.	
137A	7	18 1	31	10	1.5 &11	14.0	14.0	0.0	1.3+	Fair	

TABLE VII.

Measures and Reduction of Plate 56A *W Sagittarii.*

λ	Star	Iron	Red.	Dis.	V	λ	Star	Iron	Red.	Dis.	V
					km						km
3969.4		13.876				71.7	39.945		−3	31	29
4005.3		17.418		+	+	71.8		39.9195			
05.3	17.455		+1	32	25	75.1	40.177		−3	20	19
12.5	18.136			33	26	82.6		40.6995			
24.8	19.316			28	22	83.2	40.753		−2	30	29
41.0	20.839			45	36	90.0	41.257		−2	24	23
46.0		21.246				4290.1	41.285		−2	26	25
45.9	21.273		+1	22	18	94.3		41.5555			
63.7		22.879				94.4	41.608			26	25
63.6	22.881		±0	21	17	99.4		41.930			
67.2	23.233			22	18	4300.4	41.986			20	19
72.0	23.635			23	19	06.2	42.400			16	16
71.9		23.6125				08.1		42.5515			
4101.7	26.287			17	14	14.4	43.048			20	20
18.9	27.773			26	22	26.4	43.823			15	15
28.0	28.558			35	30	26.4		43.808			
32.5		28.882				31.2	44.180			24	24
32.7	28.935			30	26	34.7	44.395			21	21
34.5	29.112			26	22	37.8	44.680			30	30
37.0	29.325			10	9	40.7	44.869			28	28
43.6	29.861			21	18	44.1	45.110			13	13
43.9	29.896			33	29	52.0	45.639		−2	26	26
44.0		29.8665				60.0	46.130		−3	14	14
50.0	30.399			35	30	75.4	47.213			24	24
52.3	30.582			26	23	83.7		47.7665			
54.4	30.763			22	19	84.5	47.840		−3	23	24
56.8	30.955			32	28	95.0	48.567		−4	19	20
61.2	31.338			19	17	4401.2	48.912			13	14
63.6	31.547			26	23	04.9	49.188		−4	23	24
78.5	32.746			34	30	04.9		49.170			
84.7	33.198			20	18	15.3		49.840			
87.6	33.463			29	26	15.3	49.870		−5	28	29
91.7	33.790			18	16	17.9	50.037			30	31
96.0	34.143			34	31	43.0	51.673		−5	21	22
98.5	34.351			26	23	55.4	52.422		−6	19	21
98.7		34.3525				62.0	52.845		−6	33	36
4202.2		34.592				76.2		53.699		•	
02.2	34.630			27	24	94.7		54.815			
04.7	34.852			18	16	4501.3	55.240		−7	13	15
16.0	35.694		±0	21	19	71.8	59.350		−4	14	16
27.0	36.558		−1	18	17	84.0		60.015			
33.5	37.047		−2	17	16	4630.–		62.588	Mean	+	22.1km
50.6	38.363		−3	19	18	4871.9		74.436			
50.6		38.343				4891.4		75.320			
60.5		39.089				4957.7		78.172			
60.4	39.110		−3	18	17						

Table VI includes the results of the measures of the plates whose numbers on the observing list occur in column 1. Under the heading V′ the radial velocities with respect to the observer are given. All

the plates were measured without reversal with low power, in order to make the results strictly comparable with those of the standard plates. Eight of these velocities depend upon two measures, but the probable errors in column 7 depend upon one measure only, unless the results from separate lines were combined before the mean was taken. The probable errors of the first thirty-one plates, excepting 25A, were obtained from a preliminary set of reductions. Corrections for the motion of the observer are combined in column 5. The resulting values of the radial velocity of the star with reference to the Sun appear under the heading V in column 6.

Assuming the identity of the light- and velocity-periods for this star, the quantities in column 3 have been formed from the following data:

Maxima of *W Sagittarii*

1903,	August	2.865
		10.559
	September	2.243
		9.838
		17.432
		25.027
	October	2.621
1904,	June	1.649

Employing the quantities in columns 3 and 6 as abscissæ and ordinates respectively, the plot of velocity-determinations was formed which appears in Fig. 1. Ample verification of the adopted value for the period was found in the close agreement between results obtained from plates covering more than forty maxima of the curve, or an interval of 309 days. This agreement was at once evident in Table VI from a comparison of Plates 86A and 136F. But when the attempt was made to pass an elliptic velocity-curve through these observations, it was found that the plotted points oscillated above and below this curve with a period of 3.8 days, or one-half that of the light-variation. After repeated trials of various ellipses with different values of periastron time, longitude of periastron, eccentricity, maximum positive and negative velocity, and the velocity of the system, I selected the velocity-curve which is drawn in Fig. 1 with the narrower line. (The residuals from this ellipse appear under the head r' in Table VI.) In selecting this conic it was assumed

TABLE VI.

Table of Velocity Determinations From 33 Plates of *W Sagittarii*.

No.	Date G. M. T.	Interval Since Max.	V′	Reduction to Sun	V	P. E.	r′	r	Character of Spectra: Star	Character of Spectra: Comparison
	1903	d	km	km	km	km	km	km		
20B	Aug.. 3.713	0.848	−19.8	−19.3	−39.1	±0.7	+4.9	+0.7	Fair	Fair
25A	9.693	6.828	−21.0	−21.5	−42.5	±1.5	−4.2	±0.0	Fair to poor	Fair
36D.	17.678	7.219	−21.4	−23.9	−45.3	±0.6	−1.5	+0.1	Fair	Fair
46A	Sep.. 2.655	0.412	−10.8	−27.6	−38.4	±0.7	+7.4	+3.7	Fair. Overexposed	Fair
47B	4.661	2.418	−10.6	−28.1	−38.7	±0.6	−5.7	−0.4	Good	Fair
56A	7.633	5.390	+22.1	−28.4	− 6.3	±0.5	+0.6	−0.2	Fair	Good
57B	7.660	5.417	+23.8	−28.4	− 4.6	±0.8	+2.4	+1.8	Fair	Good
58A	8.631	6.388	−2.8	−28.5	−31.3	±0.6	−5.5	±0.0	Good	Good
59D.	8.651	6.408	− 4.3	−28.6	−32.9	±0.4	−5.8	−0.3	Good	Fair
60A	9.628	7.385	−15.7	−28.6	−44.3	±0.7	+0.7	+0.7	Fair to good	Good
61D.	9.643	7.400	−16.3	−28.7	−45.0	±0.7	±0.0	±0.0	Fair to good	Good
63A	10.624	0.786	−12.6	−28.7	−41.3	±0.6	+2.8	−1.4	Good	Good
64B	10.637	0.799	−11.3	−28.7	−40.0	±0.6	+4.3	+0.1	Good	Good
65A	11.620	1.782	− 9.6	−28.9	−38.5	±0.5	+0.4	−0.2	Fair	Good
66B	11.632	1.794	− 9.4	−28.9	−38.3	±0.5	+0.6	−0.4	Good	Good
75A	14.631	4.793	+21.3	−29.2	− 7.9	±0.8	+2.5	−1.4	Fair to poor	Fair to poor
76B	14.651	4.813	+23.9	−29.2	− 5.3	±1.8	+4.9	+1.0	Underexposed	Good
77A	16.620	6.782	−10.7	−29.3	−40.0	±0.5	−5.8	−0.6	Good	Good
78B	16.640	6.802	−11.1	−29.4	−40.5	±0.5	−6.0	−0.8	Fair	Good
82A	20.634	3.202	+ 3.2	−29.5	−26.3	±0.7	−0.2	+3.1	Fair	Fair
83A	21.640	4.208	+18.9	−29.6	−10.7	±0.5	+5.5	+1.0	Fair	Fair
86A	23.632	6.200	+ 4.6	−29.6	−25.0	±0.6	−2.6	+0.1	Good	Fair
91A	24.612	7.180	−11.9	−29.6	−41.5	±0.5	+2.1	+3.0	Good	Good
92B	25.617	0.590	−15.2	−29.6	−44.8	±0.7	+0.4	−3.7	Fair	Fair
93B	26.611	1.584	−10.0	−29.5	−39.5	±0.7	+0.3	−0.5	Good	Good
94D.	27.617	2.590	− 7.3	−29.5	−36.8	±0.7	−5.3	+0.2	Fair	Fair
95B	Oct. 3.615	0.994	−12.5	−29.2	−41.7	±0.5	+1.6	−2.3	Fair	Poor
99F	4.627	2.006	− 9.7	−29.2	−38.9	±0.7	−2.5	+0.1	Overexposed	Poor
100A	5.625	3.004	− 4.9	−29.1	−34.0	±0.6	−4.7	+0.4	Good	Fair
102A	6.615	3.994	+11.6	−28.8	−17.2	±0.6	+1.1	−1.4	Fair	Fair
103B	6.640	4.019	+12.4	−28.9	−16.5	±1.0	+1.8	−0.8	Underexposed	Fair
	1904									
136F	June 7.845	6.196	−31.2	+ 6.6	−24.6	±0.8	−4.6	+0.5	Good	Fair
137A	7.877	6.228	−30.3	+ 6.5	−23.8	±1.0	−2.9	+2.3	Fair	Good

that the actual observed velocities followed a superimposed curve with a period of 3.8 days and with nearly equal amplitudes for crests and troughs. For the better determination of the secondary curve the residuals of all plates from the ellipse have been plotted in Fig. 2 after being reduced to one complete period of 3.8 days, employing the well-established nodal point at 1.7 days after the light-maximum. A sine curve was then passed through these points with an amplitude of 4.2 km at the crest and 5.5 km at the trough, as shown in the diagram (Fig. 2). This final curve was superimposed on the velocity-curve and is represented by the heavy line of the upper curve of Fig. 1. The residuals for all the plates from this curve appear in Table VI under the heading *r*. Including all these residuals and assigning equal weights to each, the resulting probable error of a single plate is ±0.90 km. It can be seen by consulting the temperature-range in

Table V and the last two columns of Table VI that all the plates showing large residuals are undoubtedly inferior to the average plate. The long exposure and great temperature variation for 82A and the

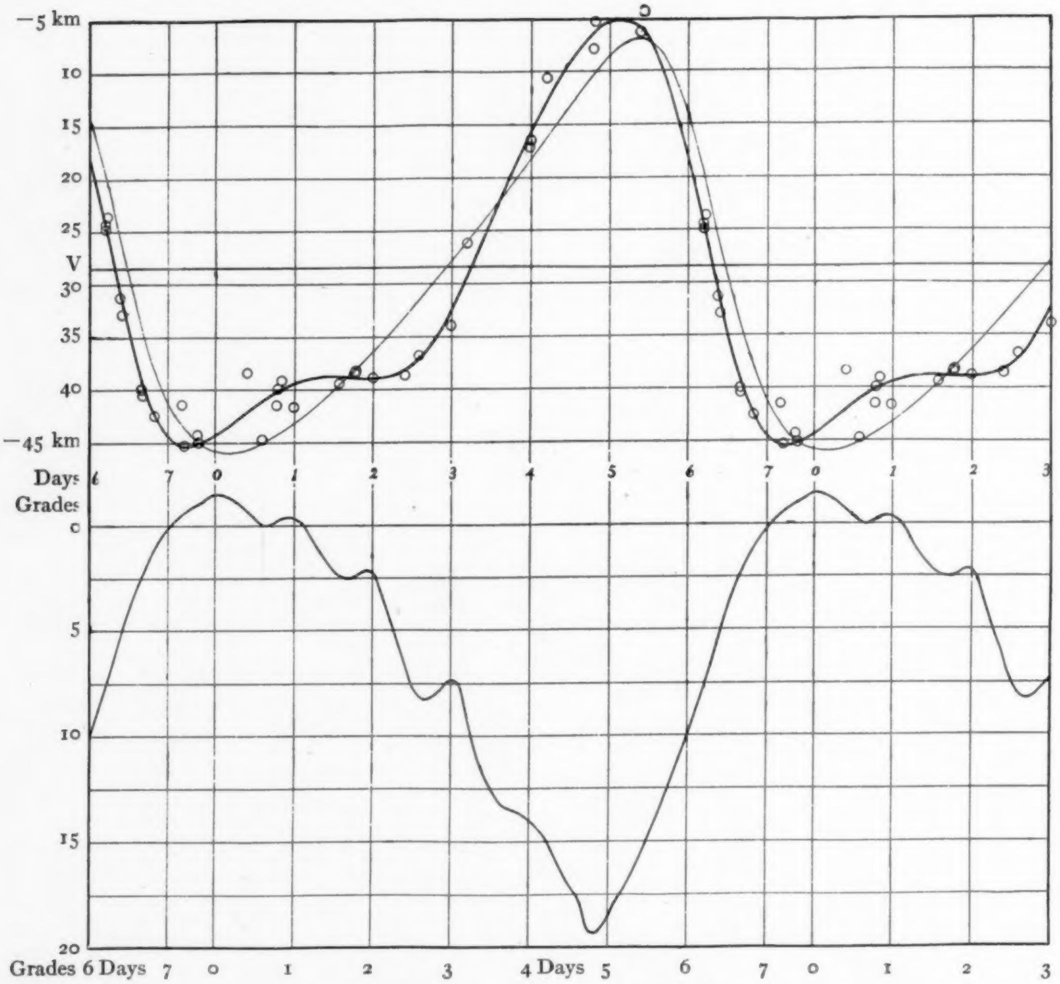


FIG. 1.—Velocity-Curves and Light-Curve of *W Sagittarii*.

poor comparison spectrum of 95 B would justify the rejection of these plates. Plate 91 A shows good spectra, but it should be noticed that its temperature-range is twice that of the average range for all the plates. Excluding five inferior plates whose velocities could be given

very small weight in the construction of the curve, the probable error of any one of the remaining twenty-eight plates is ± 0.55 km. The occurrence of relatively larger residuals along the crest of the secondary curve finds ready explanation when it is noticed that, of the two crests which obtain in one complete revolution of the system, the one occurs at the light-maximum where the greater activity of the star could lead to wide ranges in velocity, while the other occurs at light-minimum where, with increased exposure time, the effect of temperature-change in the instrument becomes maximum.

The elements of the orbital or primary curve of the brighter component of *W Sagittarii*, together with those of the superimposed secondary curve, are as follows:

TABLE VIII.

Elements.

		Primary Curve	Secondary Curve
Apparent period	P'	7.59460 days	3.80 days
Longitude of periastron	ω	70°0	
Eccentricity	e	0.320	0.0
Time of periastron after light maximum	T	6.20 days	
Projection of semi-major axis on plane of sight	$a \sin i$	1,930,000 km	
Projection of periastron distance on plane of sight	$q \sin i$	1,310,000 km	
Projection of apastron distance on plane of sight	$q' \sin i$	2,550,000 km	
Ratio of masses	$\dfrac{m^3 \sin^3 i}{(m+m_1)^2}$	0.004990	
Amplitude of velocity-curve at crest	A	+21.6 km	+4.2 km
Amplitude of velocity-curve at trough	B	−17.4 km	−5.5 km
Velocity of center of mass of the system	V	−28.6 km	

On the basis of these elements, I have constructed on Fig. 3 the elliptical orbit of the brighter component of *W Sagittarii*, indicating the line of nodes by a horizontal line and the line to the Sun by a vertical line drawn downward. Further, at the various points where they occur I have indicated the light-maximum and light-minimum and the cross-points of the primary and secondary velocity curves.

Our present knowledge of the elements of *W Sagittarii* makes possible the introduction of two small corrections to the period of this star depending upon the velocity of light. The first is due to the

change in the distance between the Earth and the bright component of the binary system occasioned by their orbital motions, and may be regarded as the combined effect of the equations of light of the

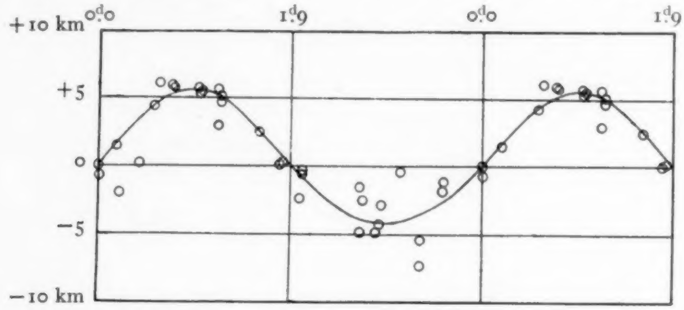


FIG. 2.—The Secondary Curve.

two bodies. Its operation is not systematic, but may be taken into account in the computed times for maxima used in plotting the velocity

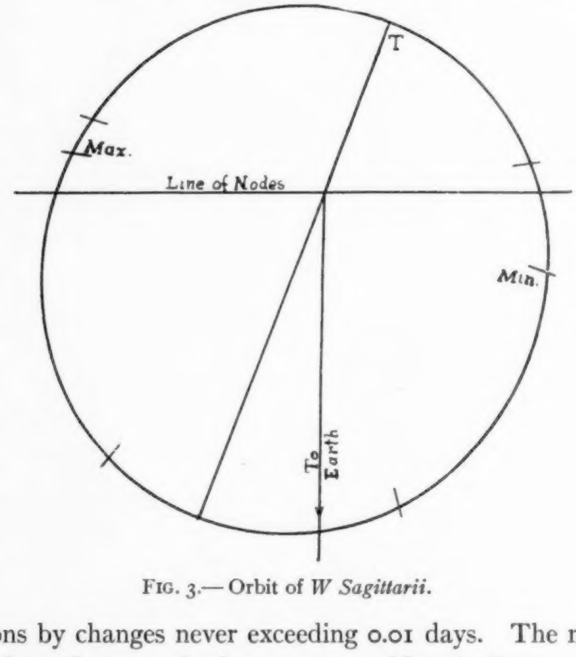


FIG. 3.—Orbit of *W Sagittarii*.

observations by changes never exceeding 0.01 days. The maximum variation from the mean in the present problem is about 0.002 days, which is equivalent to a displacement of 0.007 cm in the abscissæ of

my original velocity diagram. It is therefore evident that this correction is negligible. The second correction to the period of this variable is based upon the continual approach of the system toward the Sun at the rate of 28.6 km per second. As this correction is constant, the use of the apparent period for the velocity diagram does not affect the determination of the elements which characterize the form and position of the orbit. But the epochs of the curve referred to the time of maximum, and all dimensions of the orbit, are direct functions of the true period. The correction to the apparent period amounts to +62 seconds, or +0.00072 days. Accordingly, the true value of the light- and velocity-periods of *W Sagittarii* is

$$P = 7.59532 \text{ days.}$$

As the remaining elements of the velocity-curve are published to three places only, this change in the fifth significant figure of P does not appreciably affect their values.

The angle i, which measures the inclination of the orbital plane to a plane tangent to the celestial sphere at the center of mass of the system, cannot be determined without micrometrical measures of the two components. The absolute scale of the ellipse is therefore indeterminate. For various assumptions of i the values of q are given below.

i	Periastron Distance
15°	5,060,000 km
30°	2,620,000
45°	1,850,000
60°	1,510,000
75°	1,360,000
90°	1,310,000

In the absence of any evidence regarding the radial velocity of the fainter star it is not possible to determine the masses of the system, but the constant of attraction furnishes data for the determination of these quantities when the angle i and the ratio of the masses are known. On the basis of various assumptions for i and m_1/m, I have tabulated below (Table IX) the corresponding values of the masses, in units of the Sun's mass, of the two stars (m and m_1) together with their distance apart at perihelion (q_1+q). m_1 and q_1 refer to the bright star. The computations are based upon the equation

$$\frac{m^3 \sin^3 i}{(m+m_1)^2} = 0.00499 \odot$$

TABLE IX.

m_1	i	15°	30°	45°	60°	75°	90°
m							
	m	0.45 $\odot$	0.062	0.022	0.012	0.009	0.008
0.25	m_1	0.11 $\odot$	0.016	0.005	0.003	0.002	0.003
	$q+q_1$	6,320,000 km	3,270,000	2,310,000	1,890,000	1,700,000	1,640,000
	m	0.64 $\odot$	0.090	0.032	0.017	0.012	0.011
0.50	m_1	0.16 $\odot$	0.045	0.016	0.009	0.006	0.006
	$q+q_1$	7,590,000 km	3,930,000	2,770,000	2,260,000	2,040,000	1,960,000
	m	0.88 $\odot$	0.121	0.043	0.024	0.017	0.015
0.75	m_1	0.66 $\odot$	0.091	0.032	0.018	0.013	0.011
	$q+q_1$	8,850,000 km	4,580,000	3,240,000	2,640,000	2,380,000	2,290,000
	m	1.15 $\odot$	0.16	0.057	0.031	0.022	0.020
1.0	m_1	1.15 $\odot$	0.16	0.057	0.031	0.022	0.020
	$q+q_1$	10,120,000 km	5,240,000	3,700,000	3,020,000	2,720,000	2,620,000
	m	3.5 $\odot$	0.49	0.17	0.095	0.068	0.061
2.5	m_1	8.8 $\odot$	1.23	0.43	0.237	0.170	0.153
	$q+q_1$	17,700,000 km	9,170,000	6,470,000	5,280,000	4,760,000	4,580,000
	m	10.3 $\odot$	1.44	0.51	0.28	0.20	0.18
5.0	m_1	51.7 $\odot$	7.20	2.54	1.38	0.99	0.90
	$q+q_1$	30,400,000 km	15,700,000	11,100,000	9,060,000	8,160,000	7,860,000
	m	20.08 $\odot$	2.9	1.3	0.56	0.40	0.36
7.5	m_1	155.70 $\odot$	29.2	10.3	4.17	3.01	2.71
	$q+q_1$	43,000,000 km	22,300,000	15,700,000	12,800,000	11,600,000	11,100,000
	m	34.7 $\odot$	4.8	1.7	0.93	0.67	0.60
10	m_1	347. $\odot$	48.2	17.1	9.29	6.69	6.03
	$q+q_1$	55,700,000 km	28,800,000	20,300,000	16,600,000	15,000,000	14,400,000
	m	73.6 $\odot$	10.2	3.6	2.0	1.4	1.3
15	m_1	1104. $\odot$	153.	54.3	29.6	21.3	19.2
	$q+q_1$	81,000,000 km	41,900,000	29,600,000	24,200,000	21,800,000	21,000,000

In determining the most probable value of the true scale of the orbit from Table IX, there is opportunity for unlimited conjecture. However, in view of what is known and of what in addition will be accepted as most reasonable, the range of such speculation can be greatly diminished. In view of the fact that the spectrum of the fainter star is of vanishing order, the ratio of the light-intensities of the two stars would probably be greater than 1 to 3. Assuming, then, that the apparent surface brightness of the two stars is the same, we are led to conclude that the ratio of the masses is greater than, or equal to, 1 to 5, since this ratio varies as the three-halves power of the ratio of the apparent disks for equal densities for the two stars. We are thus quickly restricted to the last four rows in Table IX. Further the secondary curve of Plates VIII and IX possesses many features that point to its origin in the rotation of the brighter star. The varying velocity could be ascribable to a difference in brightness between two hemispheres of the star. Let us assume that the ratio

of brightness of the two hemispheres is roughly one to two and take the amplitude of the secondary curve as 5 km per second; then, on the basis of the rotation interpretation for the secondary curve of *W Sagittarii*, the radius of this body would appear to be about that of the Sun, if the inclination of the equatorial plane to the line of sight be about 90°. For other values of this inclination the corresponding values of the radius of this star, together with its mass in terms of the Sun's mass, appear in Table X, assuming equal density for the Sun and this solar-type star.

TABLE X.

i	R	m_1
15	2,700,000 km	57.5⊙
30	1,400,000	8.0
45	1,000,000	2.8
60	800,000	1.5
75	700,000	1.1
90	700,000	1.0

Reasoning from conditions that exist in our own planetary system, it seems most consistent to assume that the inclination of the equatorial plane of the brighter star to the orbital plane is approximately 0°. Comparing Tables IX and X on this basis, it will be seen that the mass-values in the latter table for each inclination would be duplicated in Table IX with a value of m_1/m equal to five or six. Confining our attention to the sixth row of Table IX, we find the inclination still undetermined. However, as the maximum orbital velocity of the smaller mass assumes the large values of 500 km per second for an inclination of 15°, 250 km for 30°, and 160 km for 45°, we naturally turn to the higher angles. I therefore suggest the following probable limiting values of the quantities involved:

Inclination - - - - - - -	i	45°–90°
Mass of brighter star - - - - -	m_1	0.9⊙ – 2.5⊙
Mass of fainter star - - - - - -	m	0.2⊙ – 0.5⊙
Periastron distance of brighter star - -	q_1	1,300,000 km – 1,800,000 km
Periastron distance of fainter star - -	q	7,000,000 km – 9,000,000 km
Radius of brighter star - - - - -	R_1	700,000 km – 1,000,000 km
Radius of fainter star	R	500,000 km – 600,000 km

As the existence of a strong variation in a star's light may be considered an exception to the general rule, it is possible that the internal conditions of this system are correspondingly extraordinary and the assumptions above are far from the truth. However, they will illustrate the outcome of one continuous train of reasoning.

The failure of double-star observers to detect any duplicity in this star leads to the conclusion that the angular distance between the components is not greater than $0''.15$. On the assumption that the greatest apparent distance between the two is 10,000,000 km, the inference may be drawn that the distance of the system from the Earth is greater than three and one-half light-years.

For purposes of comparison among spectroscopic studies of *Cepheid* variables, there exist, in addition to the present investigation, the excellent results of Wright and Bélopolsky in connection with their researches regarding the orbits of δ *Cephei*[1] and of η *Aquilae*.[2] A strong resemblance between the elements of these stars is at once evident. All show a pronounced eccentricity. Also, the time of closest approach of the bodies of the system occurs about a day before maximum brightness. The most marked analogies exist between Wright's elements of η *Aquilae* and those of *W Sagittarii;* indeed, with the exception of a small difference in eccentricity, the uncertainty of inclination, and a difference of 0.5 days in the period, the two orbits are essentially identical even in regard to the positions of principal maximum and minimum. It is interesting to notice that Mr. Wright has obtained no evidence of a secondary curve for η *Aquilae*, but such a curve would not have been recognized if its amplitude did not exceed 2 km, and in some positions no great distortion of the ellipse would occur with an amplitude of five kilometers for the superimposed curve. Referring to Mr. Wright's article, it should be noticed that the velocity-curve reproduced there is the empirical curve which was drawn for the purpose of determining the elements. His actual ellipse passes slightly below observation 11, not above it. It will also be noticed that his observations tend above the published curve from 1 day to 2.8 days after maximum and fall below from 2.8 to 4.5

[1] ASTROPHYSICAL JOURNAL, **1**, 160, 1895.

[2] *Ibid.*, **6**, 393, 1897. *Ibid.*, **9**, 59, 1899.

days. It therefore seems probable that some curve oscillating about an ellipse would represent the observations better.

Mr. Wright's observations extend over four months, and my own over a period of less than a year. It is therefore probable that any influence connected with the long-period irregularities in the light-recurrence of these stars would escape detection. Further study of these stars should reveal such phenomena.

The observed uniform correspondence between the light-variations and the orbital conditions of these three stars suggests the mutual dependence of the two phenomena. Whatever may be the cause or causes of variation in the *Geminids*, they may not operate in the same manner in the *Cepheids*. It therefore seems advisable to treat each group separately in any attempt to construct a theory for the explanation of their light-changes. I shall therefore advance some data regarding the most probable conditions that obtain in the system of *W Sagittarii*.

Darwin's expression for tidal potential is:

$$V=\frac{3m}{2r^3}\rho^2\left(\cos^2 z-\tfrac{1}{3}\right),$$

where r is the distance between the masses, ρ is the radius of the disturbed body, and z the angle between r and ρ; m is the mass of the disturbing body. Confining our attention to the line joining the two bodies, we place $z=0°$ and differentiate with reference to the direction of displacement and arrive at the expression, $F=2m\rho/r^3$, where F is the tidal force. As the ratio of apastron to periastron distance in this system is 2 to 1, it is evident that the tidal force varies in the ratio of 1 to 8 for these two positions. Using the rough conclusions as to the most probable data for the system arrived at above, we find that the intensity of the tidal force acting on the brighter body is roughly 50,000 times that of the Moon acting on the Earth. Further, by the introduction of another term usually negligible in tidal computations, we find the ratio of the intensities of the tidal forces on the side toward the disturbing body to the corresponding forces on the opposite side of the brighter star is 15 to 10 at periastron and 13 to 10 at apastron.

While the increase of light may be due to enormous tidal disruptions in the molten matter of the star's surface, accompanied perhaps with a liberation of heat from below, it should be noted that the dis-

placements of the absorption lines in the spectrum are caused by the motions of the star's atmosphere with and relative to the regions emitting the light. In the atmospheric regions above the high tidal area there would undoubtedly be an uprush of gases due to atmospheric tides, convection currents, increased light-pressure, and explosive outburst. Again, in the belt of low tide ninety degrees from the two high tides, there would probably be a corresponding recession of the atmosphere toward the star's surface. This would clearly give rise, in the actual velocity-curve, to an oscillation with respect to a true ellipse. These oscillations would fall below the mean at conjunction and above the mean at elongation, for at conjunction the high tidal areas are presented to the observer and at elongation the low tidal areas. Consulting the diagrams, this is found to be the case. We can therefore depart from the rotation theory to account for the secondary curve; and indeed this seems justifiable, for it is possible, with the operation of such immense tides through long periods of time, that the rotation periods of the stars involved should be brought to identity with the revolution period, as in case of our own satellite. Further, on the basis of the tidal theory there would not seem to be sufficient variation in brightness due to difference in intensity of the tidal force between the two hemispheres (the ratio is about 7 to 5) to produce by rotation the oscillation observed, unless we assume much greater masses for the system. The occurrence of the light-maximum when the tidal forces have fallen one-third in intensity presents an anomaly, but it is possible that the effect of these forces lags to this extent.

Returning again to the rotation interpretation, it is easy to construct a plausible explanation for the light- and velocity-curves of *W Sagittarii* on the assumption that the system is pervaded by a resisting medium which enhances the brightness of that side of the star which faces the direction of motion. Or, again, a third body might be present in the system and give rise to the perturbations observed by Schmidt in the period. Until more data are available, it would be premature to follow out such theories.

Considering all evidence, it seems reasonably certain that the star's variations in brightness, and particularly the principal variations, are attributable to the action of external forces.

It is a pleasure to acknowledge my indebtedness to Director Campbell, who placed the necessary apparatus at my disposal, and gave continual counsel and encouragement during the prosecution of the work; and to Dr. H. D. Curtis and Dr. J. H. Moore for valuable advice and assistance.

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July 1, 1904.

SOME ADDITIONS TO THE ARC SPECTRA OF THE ALKALI METALS.

By F. A. Saunders.

Lenard[1] discovered a new series and several other lines in the arc spectrum of sodium. He removed the slit of a spectroscope and in its place focused a real image of the arc. With suitable dispersion he observed that different parts of the arc gave different "lines," and the new ones which he discovered were emitted by the hottest vapor, near the positive pole. Konen and Hagenbach[2] succeeded in photographing many of these lines and finding others in the lithium spectrum which are apparently emitted under similar circumstances. The writer, also, hoped to obtain photographs of such lines, using the usual slit, if his spectrograph were designed to give very bright spectra free from astigmatism, and if an image of the proper part of the arc were cast on the slit. The attempt was made with all the elements of the lithium family and the results, which were partially successful, are given below.

The essential feature of the apparatus used was a Rowland concave grating of about 10 cm width and 305 cm radius, ruled with lines of somewhat unusual length on a parabolic surface. The ruled surface had an area of 5 by 8 cm. The grating was mounted with the slit close to it on a solid iron casting bolted to a brick wall. An arm, supported from this casting and constructed of heavy gas-pipe, carried the camera at its end and could be turned about a point immediately below the grating. The table carrying the grating turned with this arm. The light from the slit fell on a parabolic mirror, similar to the one on which the grating was ruled, placed at such a distance that the reflected light formed a parallel beam. This then fell upon the grating, and the spectra were formed about 150 cm from the grating along a curve which was nearly a circle of 75 cm radius. The incident and reflected beams at the mirror made an angle of 3° with one another, while the angle between the axis of

[1] *Annalen der Physik*, **11**, 636, 1903.

[2] *Physikalische Zeitschrift*, **4**, 592, 801, 1903.

the grating and the beam incident on it varied from 12° to 21°, depending on which order was being photographed. The axis of the grating was adjusted to meet the photographic film at its middle point. By rotation of the arm, this relation remained unchanged, but the focal distance changed with the angle. The camera was accordingly made adjustable along the normal to the grating, and by careful trial three positions were obtained at which the camera could readily be set and clamped, so as to obtain spectra in good focus whose middle points lay near λ 3900, 5900, or 7800. The whole mounting was enclosed by a wooden structure with doubly-curtained door, which the observer could enter or leave during an exposure without fogging the film.

With this arrangement the spectra were almost entirely free from astigmatism for a space of 25 cm, corresponding to nearly 2,700 tenth-meters in the first order. Economy of light was insured by the short focus of the mounting and the large size both of grating and mirror.

The spectra were photographed on films 35 cm long and usually 2.5 cm wide. These were mounted in a holder which bound the film all along both edges and forced it to take such a curvature that the spectra were in focus along its whole length. This holder could be moved vertically by measurable amounts by a couple of large screws, so that several spectra could be photographed above one another on the same film. Each film took in over 3,800 tenth-meters at once, with good definition over almost the whole length, if the adjustments were correctly made. The scale of the photographs was nearly 11 tenth-meters to the millimeter in the first order, and two sharp lines could be seen distinctly separated if they were 0.7 tenth-meters, or less, apart.

The films used were from the Seed Company and were coated with their 26 X emulsion. A few Eastman Non-Curling films were also tried and found very satisfactory for wave-lengths less than λ 5900. For photographing lines in the red, the Seed films were stained with a simple cyanin bath. The writer is glad to have an opportunity of thanking Mr. R. James Wallace of the Yerkes Observatory for the formula, furnished by him, for this staining process, and recommends it to others interested, as a very simple and efficient way of sensitizing

plates as far into the red as λ 8000 tenth-meters. Mr. Wallace's formula is as follows:

Cyanin solution in alcohol (1 : 500) - -	5 cu.cm.
Alcohol - - - - - - -	30 cu.cm.
Water - - - - - - -	60 cu.cm.
Ammonia - - - - - - -	10 drops.

The plate should be bathed in this for two minutes and washed for one. It may be used at once, even before drying. Such plates do not keep for many days as a rule, though the writer has used some that were kept under very favorable conditions for a couple of months and found them still fairly good.

The source of light for the present work was usually the carbon arc, using 10 to 15 amperes, direct current. The writer is indebted to the International Acheson Graphite Co., of Niagara Falls, N. Y., for some unusually pure graphite rods with which all these spectra were taken. This graphite by itself gave only half a dozen lines (mostly *Ca;* no iron) outside of the band spectrum of carbon (which showed the "tails" beautifully), but when it was saturated with a salt solution, several lines of titanium came out, evidently from the graphite. These were not unwelcome, as they were always sharp, and, as their wave-lengths are given in Rowland's table of solar lines, they made excellent standards of measurement. The differences in wave-length between these lines in the Sun and in the arc are too small to be worth considering in the present set of measurements. Eye observations showed that the alkali metal spectra were particularly well developed in the arc when the graphites were well saturated with salt solution and were separated by only 2 or 3 mm. With such a source the most successful photographs were taken; the well-known lines of the elements were then very much strengthened and broadened and the newer lines made their appearance. Fairly good photographs were, however, obtained with the arc longer, so that the "flames" were fully formed, if the light were taken from near the terminal. Exposures of three hours' duration were taken in the effort to pick up new lines in the deepest red. If the source of light could have been maintained in its most efficient condition during the whole time, the results might have been more complete, but the task of keeping the image of such lively flames as form these arcs constantly on the slit proved impossible.

A few photographs were also taken of the spark spectra of some of these elements, both with and without self-induction in the spark circuit, but no differences were detected in the relative intensities of any of the lines of the spectra of the spark or arc.

The photographs were usually taken with half the slit exposed directly to the light, and the other half covered with colored glass. In the resulting spectrum, lines in the first and second orders could easily be picked out where these overlap, as the ultra-violet lines were half the length of the others. In the deep-red photographs, half the slit was open for a short time and then covered with red glass, the other half being so covered throughout. In this way, images of unknown red lines would form part of the *same* spectrum with known second order lines; no shift could occur to alter their relative positions, as the colored glasses were supported independently and could be changed without affecting the position of the slit in the least. In all cases the beam of light from the condensing (quartz) lens, passing through the slit, filled the mirror and grating completely. The condensing lens was rigidly fixed throughout. The writer, for these reasons, feels certain that the photographs obtained can be relied upon to show the true positions of the lines.

The measurement of the films was accomplished by means of a Gaertner micrometer microscope with a run of 5 cm, graduated to read to 0.005 mm. Its screw was investigated and found to possess no error large enough to be worth considering. A magnifying power of about 15 diameters was used. In measuring an unknown line, in every case measurements were taken on several standard lines, lying on both sides of it, and its position was calculated from each of these; ten settings were made on each line. As a rule, the wave-length of any line, as given, is the average of several such sets of measurements taken from different photographs.

In the following tables of the complete arc spectra of the alkali elements, the writer has given in the first column the series to which the line belongs (*P* for principal, *I* for first subordinate, etc.). In the second column are placed various values for the wave-lengths and opposite each, in the third and fourth columns, the error as estimated by each observer and the observer's initial letter. The following are the observers quoted: L., Lehmann (*Annalen der*

Physik, **5**, 638, 1901); Ld., Lenard; K. and R., Kayser and Runge; K. and H., Konen and Hagenbach; H., Hagenbach (*Annalen der Physik*, **9**, 729, 1902); E. and H., Exner and Haschek (*Wellenlängen-Tabellen*, 1902); L. and D., Liveing and Dewar; B., Lecoq de Boisbaudran; and S. for the writer. The custom of stating errors seems to vary with different observers. The writer believes that the errors of measurement proper are usually small compared with errors due to wrong interpretation of the photographic image. In his own experience, several settings on a diffuse line may have agreed with one another to less than 0.1 tenth-meter, while a different observer has made equally concordant measurements leading to a result 0.2 or more away. Where so many lines are broadened or diffuse, as in these spectra, the importance of this class of error will be easily seen. The writer's own estimates of error are not based on variations in his microscope readings. If they were, they would be half or a third as large. He has tried to fix the error at such a value that the chances are extremely small that the measured wave-length will differ from the true one by more than the amount given. These estimates have been formed by the help of test-measurements taken on accurately known lines, following the same method as with unknown ones.

It is usual to give an estimate of the intensity of each line along with its wave-length. This has not been done in the following tables, as such estimates have usually, especially for the greater wave-lengths, depended more on the sensitiveness of the photographic plate for each vibration than on the real intensity in the source of light. The lines of a series, of course, decrease in intensity with decrease in wave-length; those of the principal series more rapidly than the others. The first subordinate series is stronger than the second, and the new series lines are the faintest of all. Quantitative measurements of the real intensities of spectrum lines are much to be desired, but the writer does not know of any that are applicable to these spectra.

The writer was not aware until after this work was done that Konen and Hagenbach had already found the lines at λ 6240 and λ 4148, which form a new series (with λ 4636) in the lithium spectrum. He gives his values for the wave-lengths of these and a few other lines

TABLE I.

Lithium.

Series	Wave-Length	Error	Observer	Series	Wave-length	Error	Observer
II	8127.34	0.27	L.	*I*	4132.44	0.2	K. and R.
	8127.0	0.3	S.	*II*	3985.94	0.2	"
P	6708.2	0.2	K. and R.	*III*	3924		K and H.
III	6240.8		K. and H.		3921.8		E. and H.
	6240.3	0.4	S.	*I*	3915.2	0.2	K and R.
I	6103.77	0.03	K. and R.	*II*	3838.3	3.0	"
II	4972.11	0.1	"	*I*	3794.9	5.0	"
III	4636.14		H.	*I*	3718.9	5.0	"
	4636.04		K. and H.	*I*	3670.6	5.0	"
	4636.3	0.4	S.	*P*	3232.77	0.03	"
I	4602.37	0.1	K. and R.	*P*	2741.39	0.03	"
	4603.04	0.01	H.	*P*	2562.60	0.03	"
	4602.00		H.	*P*	2475.13	0.1	"
	4603.2	0.2	S.	*P*	2425.55	0.1	"
	4601.6	0.2	S.	*P*	2394.54	0.2	"
II	4273.44	0.2	K. and R.	*P*	2373.9		L. and D.
III	4149.1		K. and H.	*P*	2359.4		"
	4148.2	1.0	S.				

in the hope that they may be of value, especially as some of them differ by considerable amounts from the values already given. The line at λ 4148, such as it is, is visible in Fig. 6, Plate I of Kayser's *Handbuch*, Vol. II, immediately to the right of λ 4132.44 (which by a misprint is numbered 4273 in the figure).

The "line" at λ 4602 deserves especial mention. Kayser (*Handbuch*, Vol. II, p. 366; also Fig. 5, Plate II) regards this as a line heavily reversed and much broadened towards the red. Hagenbach regards it as a pair of lines, the weaker one being constantly reversed, the heavier occasionally; the separation of the two amounts to over a tenth-meter. He is, apparently, ready to believe that all the lines in this spectrum are pairs with so great a separation. It is easy to obtain photographs of the strong line at λ 6708, and others, in which the lines are sharp enough to show a doubling, if the components were even closer than a tenth-meter, and no such doubling has been observed. The curious reversals obtained by Hagenbach certainly call for an explanation, but it is difficult to adopt the one given by him in view of the absence of doubling in those lines where it would be most easily detected. The writer took a set of six photographs of the 4602 group on a film, using the second order spectrum (5.5

tenth-meters to the mm), and varying the exposures and amounts of vapor in the arc so as to furnish wide differences among the set. In these photographs, the points of maximum density in the image on either side of the "reversal" remain in constant positions, even though the amount of vapor in the arc is small, in which case they are separated from each other by an absolutely clear space on the negative. The conclusion seems unavoidable from these images that we have here to deal with no reversal at all, but with two lines: a strong one at λ 4603.2, much broadened toward the red, and a weaker one at λ 4601.6, broadened toward the violet, neither of them being ordinarily reversed. If this view be adopted, the spectrum of lithium shows another analogy to that of sodium, for, in the latter spectrum, immediately beside the pair in the first subordinate series which is homologous to *Li* 4602 lies a faint pair broadened toward the violet in a similar manner.

The 4602 group presents the same aspect in the spark spectrum as in the arc.

In the spectrum of sodium, the new series of Lenard has been successfully photographed and measured; a new term in the red has been added and a faint haze at λ 4372 was detected on one photograph which is doubtless the sixth member of the series. The pair at λ 4472 was so diffuse that the lines could not be seen separately; the setting was made on the middle of it. The writer's measurements differ considerably from those of Lehmann and of Konen and Hagenbach on several lines; a repetition of the measurements led to the same values. Konen and Hagenbach give a line at λ 4973 which does not appear with certainty on any of the present photographs; nor does there seem to be any companion to λ 4660. The pair at λ 7410 is exceedingly faint, and may possibly not belong to sodium. It has, however, approximately the same separation as the Lenard series pair near by, and, like that pair, the line of greater wave-length is slightly the stronger. If this is a member of another series, it would seem likely that the other lines are too faint to have been observed, unless, possibly, it could be grouped with the pair at λ 5670 and the lines at 4975 and 4660. An extremely faint group was observed at λ 8210, but it was impossible to determine just what it was; this was the greatest wave-length which was photographed on these films.

TABLE II.

Sodium.

Series	Wave-Length	Error	Observer	Series	Wave-Length	Error	Observer
I	8194.76	0.2	L.		4820		Ld.
	8196.1	0.4	S.	*II*	4752.19	0.15	K. and R.
I	8184.33	0.2	L.	*II*	4748.36	0.15	"
	8184.5	0.4	S.		4730		Ld.
	7418.3	0.4	S.	*I*	4669.4	0.5	K. and R.
	7410.0	0.4	S.	*I*	4665.2	0.5	"
III	7377.4	0.4	S.		4660		K. and H.
III	7369.4	0.4	S.		4660.2	0.5	S.
II	6161.15	0.1	K. and R.	*III*	4633.1		K. and H.
II	6154.62	0.1	"		4629.5	1.0	S.
P	5896.16			*III*	4629.4		K. and H.
P	5890.19				4625.5	1.0	S.
I	5688.26	0.15	"	*II*	4546.03	0.2	K. and R.
I	5682.90	0.15	"	*II*	4542.75	0.2	"
	5675.92	0.15	"	*I*	4500.0	1.0	"
	5670.40	0.15	"	*I*	4494.3	1.0	"
III	5531.7		K. and H.	*III*	4470		Ld.
	5532.7	0.4	S.		4472.5	2.0	S.
III	5527.1		K. and H.	*II*	4423.7		L. and D.
	5528.2	0.4	S.	*II*	4420.2		"
II	5153.72	0.1	K. and R.	*I*	4393.7		"
II	5149.19	0.1	"	*I*	4390.7		"
	5100		Ld.	*III*	4372	5.0	S.
I	4983.53	0.2	K. and R.	*II*	4343.7		L. and D.
I	4979.30	0.2	"	*I*	4325.7		"
	4976.1		K. and H.	*P*	3303.07	0.03	K. and R.
	4975.0	0.4	S.	*P*	3302.47	0.03	"
	4973.0		K. and H.	*P*	2852.91	0.05	"
III	4913.5		"	*P*	2680.46	0.1	"
	4918.4	1.0	S.	*P*	2593.98	0.1	"
III	4910.1		K. and H.	*P*	2543.85	0.1	"
	4914.0	1.0	S.	*P*	2512.23	0.2	"

The lines λ 5100, 4820, and 4730 mentioned by Lenard could not be found on the photographs, nor were eye observations with a plane grating any more successful.

It may be worth noting that the wave-number differences for Lenard's series are as follows: 14.72, 14.77, 18.2, and 18.7. As the first two of these are probably accurate to one part in 80, while the last two are much less accurate, it seems fair to say that these differences are probably much smaller than the value for the usual series (17.2).

Many unsuccessful attempts have been made to find a simple formula which would express Lenard's series. The formulæ of Kayser and Runge, Rydberg, Fowler and Shaw[1], and Ritz[2] and modi-

[1] ASTROPHYSICAL JOURNAL, **18**, 21, 1903.

[2] *Annalen der Physik*, **12**, 264, 1903.

fications of these have been tried. Unless the formula contained four adjustable constants, it could not be made to fit the observations with any degree of precision. As almost any series, if not too accurately known, could be represented by a four-constant formula, no results of this work are given.

TABLE III.
Potassium.

Series	Wave-Length	Error	Observer	Series	Wave-Length	Error	Observer
	7931.8	0.5	S.		4863.8		L. and D.
	7699.3	5.0	K. and R.	*II*	4864.5	0.8	S.
P	7697		R.		4862		R.
	7701.92	0.52	L.		4856.8		L. and D.
	7699.08	0.3	S.	*I*	4856.8	0.8	S.
	7665.6	5.0	K. and R.		4857		R.
P	7664		R.	*II*	4850.8		L. and D.
	7668.54	0.52	L.		4851.0	0.8	S.
	7664.91	0.3	S.		4829		R.
I	6966.3	0.4	S.		4808.8		L. and D.
	6938.8	0.5	K. and R.	*I*	4803.8		"
II	6939		R.		4803		R.
	6939.5	0.4	S.	*II*	4801		R.
	6911.2	0.5	K. and R.		4796.8		L. and D.
II	6913		R.		4798		R.
	6911.8	0.4	S.		4788.8		L. and D.
I	5832.23	0.05	K. and R.		4767		R.
I	5812.54	0.05	"	*I*	4759.8		L. and D.
II	5802.01	0.05	"		4760		R.
II	5782.67	0.05	"		4642.35	0.3	R.
I	5359.88	0.15	"		4642.5	0.3	S.
I	5343.35	0.15	"		4638.6		R.
II	5340.08	0.15	"	*P*	4047.36	0.03	K. and R.
II	5323.55	0.15	"	*P*	4044.29	0.03	"
I	5112.68	0.2	"	*P*	3447.49	0.03	"
II	5099.64	0.2	"	*P*	3446.49	0.03	"
I	5097.75	0.2	"	*P*	3217.76	0.03	"
II	5084.49	0.2	"	*P*	3217.27	0.03	"
I	4965.5	1.0	"	*P*	3102.37	0.1	"
II	4956.8	1.0	"	*P*	3102.15	0.1	"
I	4952.2	1.0	"	*P*	3034.94	0.1	"
II	4943.1	1.0	"	*P*	2992.33	0.15	"
	4870.8		L. and D.	*P*	2963.36	0.2	"
I	4871.3	0.8	S.	*P*	2942.8	1.0	"
	4870		R.				

In the spectrum of potassium, two new lines were found, one in almost the position predicted for it (see Ritz, *loc. cit.*). It is one of the hitherto missing first pair of the first subordinate series, which is, for some obscure reason, very faint. (Ritz has given reasons for believing this to be the first, rather than the second, subordinate

TABLE IV.

Rubidium.

Series	Wave-Length	Error	Observer	Series	Wave-Length	Error	Observer
	8513.26	0.26	L.		5259.8		B.
P	7950.46	0.32	L.	*I*	5260.51		R.
	7947.6	0.5	S.		5260.5	0.4	S.
	7805.98	0.54	L.	*II*	5234.6		R.
P	7799		R.		5234.0	0.7	S.
	7800.2	0.5	S.		5194.8		B.
Sat.	7759.5	0.5	S.	*I*	5195.76		R.
I	7753.58	0.54	L.		5195.9	0.5	S.
	7757.9	0.5	S.		5165.35		R.
I	7626.66	0.32	L.	*II*	5171	2.0	S.
	7619.2	0.3	S.		5161.8		B.
II	7406.19	0.25	L.	*I*	5151.20		R.
	7408.5	0.4	S.		5150.8	0.5	S.
II	7277.01	0.25	L.	*II*	5132		R.
	7280.3	0.3	S.		5133.5	0.8	S.
	6306.8		R.		5085.8		B.
I	6298.7	0.2	K. and R.	*I*	5089.5		R.
	6298.8	0.3	S.		5088.6	0.6	S.
I	6206.7	0.2	K. and R.	*I*	5076.3		R.
	6206.7	0.3	S.		5075.7	0.6	S.
II	6159.8	0.2	K. and R.		5037		R.
	6160.0	0.3	S.		5021.8		B.
II	6071.2	0.2	K. and R.	*I*	5023		R.
	6071.1	0.3	S.	*I*	5017		R.
I	5724.41	0.15	K. and R.	*I*	4983		R.
II	5654.22	0.15	"		4967		R.
I	5648.18	0.15	"	*P*	4215.72	0.03	K. and R.
II	5579.3		R.	*P*	4201.98	0.03	"
	5579.4	0.4	S.	*P*	3591.74	0.05	"
I	5431.83	0.15	K. and R.	*P*	3587.23	0.05	"
	5431.9	0.4	S.	*P*	3351.03	0.05	"
II	5391.3		R.	*P*	3348.86	0.05	"
	5391.2	0.4	S.	*P*	3229.26		R.
I	5362.94	0.2	K. and R.	*P*	3228.17		R.
	5363.1	0.4	S.	*P*	3158.7	0.3	S.
II	5322.83		R.				
	5323.1	0.5	S.				

series, as Kayser and Runge classified it.) The other member of this pair was not found, owing to the broadening of the line at λ 6939, near which it doubtless lies. The other new line (λ 7931.8) was so faint that its companion, if it is a member of a pair, could not be seen. The first term of the principal series was excellently photographed on several films and the writer feels considerable confidence in the value of the wave-lengths given for these lines. They are recorded to the second place of decimals, as the difference between the values

could be determined more accurately than the values themselves. The best measurements were taken on photographs where the potassium was present as an impurity and the lines were fine and sharp, though measurements on heavily reversed images of these lines gave concordant results.

The line at λ 4642 seems to be outside the series formation, and, along with the faintness of the first subordinate pair, offers a very odd peculiarity in this spectrum.

Three new lines were found in the spectrum of rubidium. One is a line at λ 3158 (not seen as a pair) belonging to the principal series; another, a very diffuse and faint line at λ 5171, which, with λ 5234, forms a pair in the second subordinate series. The third is a line at λ 7759.5 which is a companion to the first subordinate series line λ 7757.9. The results of the measurements in the deep red differ from those of Lehmann to a marked degree. As a check on the present methods, the writer took a set of measurements of several other and better-known lines in this spectrum. The results, which are included in the table, agree reasonably well with those of Kayser and Runge. As a further check, the wave-number differences for the pairs were calculated, using the writer's values for the wave-lengths. The results were:

P Series	Subordinate Series I	Subordinate Series II
237.8	234.7	237.7
	235.6	237.7
	235.8	237.1
	236.2	237.3
	236.3	232.8
	233.5	

The constancy of these numbers is satisfactory, with the exception of the short wave-length ends of the series, where the measurements do not pretend to be accurate. A similar table calculated from Lehmann's results shows a much wider divergence.

Ramage has noted a line at λ 6306; nothing was seen at this place except a ghost of the strong line at λ 6299. He has also given a sharp line at λ 5165; at this point on the writer's photographs appeared a sharp line which was the head of a carbon band.

TABLE V.

Caesium.

Series	Wave-Length	Error	Observer	Series	Wave-Length	Error	Observer
I	9211.86	0.7	L.	*Sat.*	6217.6		R.
IV?	9171.88	0.7	L.		6217.6	0.3	S.
P	8949.92	0.76	L.	*I*	6213.4	0.5	K. and R.
I	8766.10	0.32	L.		6213.1	0.3	S.
P	8527.72	0.32	L.	*II*	6034.43		R.
III	8080.02	0.48	L.		6034.8	0.3	S.
III	8019.62	0.48	L.	*I*	6010.59		R.
	8007.1	0.5	S.	*Sat.*	5847.86		R.
II	7944.7	0.3	S.	*I*	5845.31		R.
II	7616.58	0.44	L.	*II*	5839.33		R.
	7609.7	0.3	S.	*II*	5746.37		R.
IV	7280.5	1.0	S.	*I*	5664.14		R.
IV	7227.46	0.44	L.	*I*	5635.44		R.
	7228.8	1.0	S.	*II*	5574.4		R.
Sat.	6984		R.	*II*	5568.9		R.
	6983.8	0.3	S.	*I*	5503.1		R.
I	6973.9	5.0	K. and R.	*I*	5466.1		R.
	6973.1	0.3	S.	*I*	5414.4		R.
III	6869		R.	*II*	5407.5		R
	6872.6	1.0	S.	*I*	5351		R.
III	6829		R.	*I*	5341.15		R.
	6826.9	1.0	S.	*I*	5304		R.
I	6723.6	5.0	K. and R.	*I*	5256.96		R.
	6723.7	0.2	S.		5209		R.
IV	6630		R.	*I*	5199		R.
	6630.5	1.0	S.	*I*	5154		R.
IV	6588.0	1.0	S.	*P*	4593.34	0.05	K. and R.
II	6590		R.	*P*	4555.44	0.05	"
	6587.3	0.3	S.	*P*	3888.83	0.1	"
III	6472		R.	*P*	3876.73	0.1	"
	6475	2.0	S.	*P*	3617.08	0.3	"
III	6433		R.	*P*	3611.84	0.2	"
	6434	2.0	S.	*P*	3477.25		R.
IV	6359	3.0	S.	*P*	3398.40		R.
II	6354		R.	*P*	3348.72		R.
	6355.3	0.3	S.	*P*	3314		R.
IV	6325	3.0	S.	*P*	3287		R.

In the caesium spectrum, five new lines were found. The one at 7944.7 was predicted by Ritz and is a member of the second subordinate series, along with 7609.7.

Two new series can be arranged from the odd lines in this spectrum, which might be called the third and fourth subordinate series, as they evidently belong to this class.

Subordinate Series III	Subordinate Series IV
8080	?
8019	9171 ?
6872.6	7280.5
6826.9	7228.8
6475	6630.5
6434	6588.0
	6359
	6325

The wave-number differences of these pairs are approximately constant; they are: 96, 97.5, 97.7, 98.5, 98.4, and 80. The last pair, being on the verge of invisibility, is very inaccurately measured. These differences are much less than for the usual series, being about 0.177 as much. The line of greater wave-length in these pairs has slightly greater intensity. All the lines in these series are so exceedingly diffuse that their positions cannot be measured with much accuracy; it did not seem worth while on this account to attempt to fit a formula to them. From an inspection of their positions, it seems likely that they are not so directly connected with the first and second subordinate series as in the case of Lenard's series in the sodium spectrum. They appear to run together to a common end, which lies somewhat nearer the red than the end of the other series. It seems most unlikely from their appearance that they can be due to any impurity, as in that case they might fairly be expected to be sharp when faint; they are, however, always diffuse. These series are faint, and of approximately equal intensity.

It does not appear to have been previously noticed that the heavier line of each pair in the first subordinate series is accompanied by a satellite (lines all observed by Ramage) on the red side, forming a "secondary" series, using Rydberg's term. The writer has calculated the wave-number differences of the pairs in the caesium spectrum, using Lehmann's data for the two extreme pairs and his own for most of the rest. The results are as follows:

P Series	Subordinate Series I	Subordinate Series II
553.2	552.0	554.1
	531.9	554.2
	542.3	553.7
	547.2	554.6
	549.7	

It will be noticed that the values for the first subordinate series are lower than the others, and have an evident drift, with the exception of the first, which is possibly in error. As the first member of each pair is accompanied by a satellite (none have yet been observed for the first and last lines), the wave-number differences were calculated between the satellite and the line of shorter wave-length. The result is shown below:

Wave-Number Differences Calculated from Main Line	Wave-Number Differences Calculated from Satellite
531.9	553.7
542.3	553.9
547.2	554.7

It will be seen at once that the differences calculated from the satellites agree beautifully with those of the second subordinate series. Apparently, then, the wave-number difference for the first subordinate series, as usually calculated, should vary slightly, increasing with decreasing wave-length. A glance at the table of differences for rubidium shows the same effect in that spectrum also, and there, too, if we use the satellite at λ 7759.5 (the only one yet observed) with the line 7619.2, we obtain a difference of 237.3 instead of 234.7, thus bringing this value into agreement with those for the second subordinate series. This principle may be used to calculate the positions of the other satellites in the rubidium spectrum. They should be at λ 9213.6 (using Lehmann's line 8766.10 for the calculation), 6299.6 and 5725.1, thus being within very short distances of their parent lines. As the latter are broadened toward the greater wave-lengths they cover up the satellites, so that these have not yet been observed. The satellites in the spectra of lithium, sodium, and potassium are also, probably, too close to their parent lines to be distinguishable.

The writer wishes to express his indebtedness to the committee in charge of the Rumford Fund for a grant covering the expenses of this investigation.

SYRACUSE UNIVERSITY,
June, 1904.

ON THE INVESTIGATION OF SIMULTANEOUS OCCURRENCES IN THE SOLAR ACTIVITY AND TERRESTRIAL MAGNETISM.

By A. NIPPOLDT.

AFTER a long period of purely statistical investigation of the connection between terrestrial magnetism and solar activity, an advance has been made in recent years by examining this relation in detail, *i. e.*, by investigating how a definite magnetic disturbance is associated with centers of solar activity simultaneously present. But in so doing it does not seem to be quite possible to entirely escape from the statistical method of the earlier date, and false conclusions have accordingly been reached. The present paper is intended to demonstrate this, with a special reference to the important studies of Father Cortie. It will be done by the discussion of a special case, to which Cortie assigns the greatest weight, and from which he reaches results entirely different from those of the writer.

In this JOURNAL[1] there recently appeared a paper by Father Cortie, entitled "On the Solar Prominences and Terrestrial Magnetism," in which he sought to prove from the magnetic records at Stonyhurst that a vigorously active Sun-spot had no effect on terrestrial magnetism. A paper in *Monthly Notices*[2] had a similar object; and an earlier investigation of the three years of minimum, 1899–1901,[3] is closely related with these. So far as known by the writer, Cortie's results have not yet been contradicted, and have already been cited in different places as undisputed, *e. g.*, by Deslandres.[4]

A powerful objection to any causal relationship between solar activity and terrestrial magnetism would be found in Cortie's attempted demonstration that there once occurred a Sun-spot of marked activity and lasted for two rotations of the Sun, which fell at

[1] ASTROPHYSICAL JOURNAL, **18**, 287–293, 1903.

[2] **62**, 516–521, 1902.

[3] ASTROPHYSICAL JOURNAL, **16**, 203–210, 1902.

[4] *Comptes Rendus*, **137**, 822, 1903.

a time of the greatest quiet and was accompanied by no magnetic disturbance. The following investigation is therefore devoted to this special case, the Sun-spot of May 19–June 26, 1901.

Cortie summarizes his result, so far as it concerns our question, in the following way: "that in the case of the only great Sun-spot of an otherwise absolutely quiet year, there is no connection between the solar storm and magnetic disturbances."[1] Three pages previously he established solely that "the magnets did not show even a moderate disturbance." As valuable as Cortie's service is, in that he gives greater importance to the investigation on the individual Sun-spot than to the statistical method, he nevertheless cannot quite emancipate himself from the latter, and he again introduces an idea so distinctly statistical as that of a "moderate disturbance." This idea had been previously developed by Ellis,[2] who, chiefly for statistical purposes, had to undertake some kind of a classification. Such a standard is, however, for the most part of value only for the purpose for which it was made, *i. e.*, here only for statistical ends. As is well known, Ellis chose the maximum amplitude as the basis for his classification of disturbances, and accordingly the disturbance of May 31, 1901, may actually not have been "a moderate disturbance." At Potsdam the maximum amplitude in declination was only $5'.4$, which is less than that of $10'$ fixed by Ellis as the lower limit of a moderate disturbance. The amplitude in horizontal intensity amounts, however, to 84×10^{-5} C.G.S. units, therefore exceeding by 34γ ($1\gamma = 1 \times 10^{-5}$C.G.S.) the lower limit for a moderate disturbance. We have 23γ as the maximum amplitude of vertical intensity. Therefore the disturbance at Potsdam must be reckoned at least as a moderate one. The schedule of Ellis fits only for places of similar geographical situation to Greenwich; at other places different upper and lower limits must be fixed.

The maximum amplitude can, indeed, hardly be usable for special investigations, in order to decide whether or not a curve is disturbed. One and the same magnetic disturbance may produce at one place a large, and at another a small maximum amplitude. For this we can only compare the registrations on the same day at different places.

In order to escape from the accidental character of numbers of this

[1] *Monthly Notices*, **62**, 521, 1902. [2] *Ibid.*, **60**, 142–157, 1899.

sort, another sort of classification—in five characters—was introduced by Eschenhagen, which has been adopted by many observatories in their regular publications. This classification is based on the whole general appearance presented by the curves registered: the more irregular it appears, the more disturbed is the course of the elements.[1]

Employing this method, the writer would say: *A disturbance occurs when the whole character of the variation becomes different from what it was before.*

Nothing illustrates this better than just this disturbance of May 31, 1901, now under discussion.

The variation in the horizontal intensity is shown in the accompanying figure, which was made from a contact photograph and was not in any way retouched.[2] The scale of time is omitted, because we are not concerned with it here. One mm in vertical distance corresponds to 9.73 γ. The original records in declination and vertical intensity are unfortunately too weak for reproduction.

The illustration begins at about midnight of May 30, and ends with the following midnight of May 31. The curves for all three elements have an entirely normal course until about the middle of the forenoon. Then there suddenly occurs at $9^{h}\ 13^{m}.5$ A. M., Potsdam Mean Time $= 8^{h}20^{m}.7$ G.M.T., a typical outbreak of disturbance, and from this time on the whole character of the variation is entirely different from what it was before. There cannot be the slightest doubt that something has happened to the terrestrial magnetism which renders the normal course of the curve impossible. We must designate this condition as a disturbance, since it is of such an entirely different order from the preceding normal condition, even if the maximum amplitude in declination did not attain the value demanded by Ellis's scale. Now, since the

FIG. 1.—Curve of Magnetic Declination at Potsdam, May 30–31, 1901.

[1] (A more precise definition is given in the *Results of Magnetic Observations at Potsdam in 1890 and 1891.*)

[2] It has been necessary to reduce the illustration to one-third the size of the photographic print sent by the author.—EDS.

summary published by Cortie himself for the remaining days of the interval while the Sun-spot lasted indicated magnetic unrest frequently, we must unquestionably conclude that *at the time when that large Sun-spot was present, the terrestrial magnetism exhibited quite a number of disturbed days along with quiet days, and that therefore the conclusion cannot be drawn "that there was no connection between the solar storm and the magnetic disturbance."*

But even if no departure from the normal course of the terrestrial magnetism had been observed either at Stonyhurst or at Potsdam, or at any other observatory in middle and lower latitudes, the proof would nevertheless have to be supplied that no disturbances were observed also in high latitudes. *We could consider it as proved that a center of activity on the Sun was without influence on the terrestrial magnetism only when stations near the pole also exhibited no deviations from their normal curves.* Without committing ourselves to any one of the many new conceptions as to the nature of the effect of the solar activity upon terrestrial magnetism, we may be permitted to represent it as a sort of relay action: the strength of the releasing solar activity need not have a definite relation to the strength of the magnetic storm.[1] Thus, for instance, the large spot of October 12, 1903, was accompanied by only a relatively small magnetic perturbation, while the decidedly smaller spot of October 31 was associated with the largest disturbance ever observed. (At Potsdam the maximum amplitude and declination was $3^\circ 10'$, in horizontal intensity 950γ, in vertical intensity 960γ). The magnitude of the disturbance therefore cannot be decisive for our question.

Hence it was insisted, in what has preceded, that in all investigations of the relation between solar activity and magnetic variations there should not be any kind of statistical definition of the idea of disturbance, but the appearance of the curve should decide whether or not a disturbance occurred. A strong element of personality therefore enters into the matter. The force of this objection is somewhat diminished by the fact that each reader can form his own judgment from the illustrations accompanying any article or from the observed data at his disposal. If the case under consideration is a

[1] See also the attempted explanation by E. W. MAUNDER in *Monthly Notices*, **64**, 222–224, 1904.

sudden outbreak of disturbance, then at most only the time when the disturbance ended is uncertain. In other cases it might easily be possible for different investigators to be of different opinions. But this is also just the condition in regard to centers of solar activity. A gradual transition occurs between the large faculæ and the ordinary gaseous eruptions, and similarly there is a whole series of intermediate steps between the different courses of the magnetic variations. Indeed the automatic records of such sensitiveness as that attained by Edler at the Potsdam Observatory have revealed the fact that even an entirely normal curve is constituted of very numerous, though very small, oscillations; and that therefore here also, just as in the case of disturbances on a large scale, a force in violent variation discloses itself as the effective cause. This the author would associate as of the first importance with the uninterrupted activity of the Sun when free from spots.[1]

We therefore substitute for the statistical method, which can hardly furnish us with anything new, the investigation in detail; that that is, we make a study of each individual disturbance for itself in all its peculiarities. But it will not be possible to fully answer the questions as to the nature of the disturbances unless assistance is also offered by the astrophysicist to the magnetic observer. We must know what happened on the Sun at the time when definite phenomena occurred in the magnetic variations. As was recently shown by Deslandres,[2] the surveillance of the Sun as now maintained is inadequate for the purpose. The desired information can be supplied only by a continued and uninterrupted registration of the actions occurring on our central body.

Magnetisches Observatorium, Potsdam.
July 7, 1904.

[1] Further particulars will presently be given in the *Meteorologische Zeitschrift*. See also the *Verhandlungen der Gesellschaft Deutscher Naturforscher und Aerzte zu Cassel*.

[2] *Comptes Rendus*, **137**, 821–827, 1903.

PRELIMINARY COMMUNICATION ON THE INFRA-RED ABSORPTION SPECTRA OF ORGANIC COMPOUNDS.

By William W. Coblentz.

The investigation of absorption spectra far into the infra-red has never been made in a thoroughly systematic manner. This is no doubt due to the enormous difficulties to be encountered and the slowness with which observational data can be obtained, so that usually after investigating half a dozen compounds, the results have been given to the public. As a consequence the agreement in the location of certain absorption bands is not very satisfactory, while some of the conclusions arrived at are not always convincing. This is partly due to the fact that the infra-red spectrum has never been examined farther than 7 μ for alcohols[1] and to about 10 μ for several other compounds. Now, it so happens, as will be shown later on, that with the limited dispersion at our disposal (if that be the true reason) all carbohydrates investigated show a large absorption band between the wave-lengths 3 μ and 3.5 μ, and then there are no marked bands till we arrive at about 7 μ. Beyond this, to the limit of the working transparency of rock-salt at 15 μ, there are numerous sharp, well-defined bands. Take then, for example, the question of the influence of the chemical structure of the molecule on absorption. Julius,[1] using a rock-salt prism, investigated about twenty compounds and found the absorption of isomeric alcohols quite similar from 3 μ to 7 μ.

Puccianti,[2] using a quartz prism to 2.5 μ, found the three isomeric xylenes so similar that a critical examination of his curves is necessary to convince one that structure influences absorption. The same is true of my own work on normal and iso-caproic acid, in which for the region from 3 to 6 μ the curves are identical. In general, it is only after one arrives at 8 to 12 μ that new bands occur. Of all the compounds studied, the only exception to the above statements is that of the mustard oils, *R-NCS*, and the sulphocyanates, *R-SCN*.

[1] *Verhandl. Konikl. Akad., Amsterdam*, Deel I, No. 1, 1892.

[2] *Nuovo Cimento*, **11**, 241, 1900.

As will be pointed out later on, the mustard oils have an enormous absorption band at about 4.78 μ which occurs as a slight band at about 4.68 μ in the sulphocyanates, and is to be found in no other compounds, except as a moderately strong band at 4.6 μ in carbon disulphide.

This past year has been occupied, under an appointment as research assistant by the Carnegie Institution of Washington, in exploring the absorption spectra of at least one hundred and twenty compounds of carbon and hydrogen. Of this number about one hundred liquids or solids and fourteen gases were explored to 14 μ, using a rock-salt prism, while nineteen liquids were explored to 2.5 μ, using a quartz prism.

These data are as yet only incompletely worked up, and since there will be considerable delay in making the complete report, it has seemed wise to communicate this preliminary note. The problem before me was to determine the effect of molecular weight upon absorption; also the effect of chemical structure, *i. e.*, the arrangement of the atoms in the molecule, and the effect produced by the substitution of a CH_3 or OH group of atoms.

As a criterion for the effect of the substitution of a CH_3 group, the conspicuous band occurring between the wave-lengths 3.0 μ and 3.5 μ was critically examined. Julius found this band at 3.45 μ for compounds containing CH_3 groups, and hence ascribed it to this group. As a standard for judging the effect of the OH radical in certain compounds, the water bands found by Aschkinass[1] at 3 μ and 6 μ were selected. Ransohoff,[2] in his study of several alcohols, had tacitly concluded that the band at 3 μ was due to the OH radical. Such conclusions in regard to the CH_3 and OH groups seemed contradictory to the work of Ångström and Palmer,[3] who found that the Cl band at 4.28 μ does not occur in the six compounds investigated by Julius. The latter had previously shown that the chemical atom lost its identity in a compound, so that one cannot foretell the absorption spectrum of a compound from a knowledge of the spectra of the constituent elements. In addition to this we have the phenomena observed of solutions, in which the solute, sulphur[4] in carbon disulphide

[1] *Annalen der Physik*, **55**, 401, 1895.
[2] *Inaug. Dissertation*, Berlin, 1896.
[3] *Öfversigt Kongl. Vet. Akd.*, No. 6, 389, 1893.
[4] JULIUS, *loc. cit.*

and iodine[1] in carbon disulphide and chloroform, loses its absorbing power in the infra-red, and does not affect the selective absorption of the solute. In the optical region, however, the solution has a strong absorption band, which would indicate a resonance of small particles, as distinguished from the intra-molecular resonance of the solvent. The distinction between solvent and solute does not apply to mixtures of gases. The absorption spectrum of a mixture of CO and CO_2 is composed of the spectra of the separate gases. The same is true of the spectrum of illuminating gas, which is the composite of CO, CO_2, CH_4, C_2H_4, etc. From this it must not, however, be assumed that the gases are unique, for ethylene chloride shows bands belonging to carbon tetrachloride which it contains as an impurity, while toluene shows absorption bands common to thiophene, also present as an impurity.

The apparatus used in this work consisted of a 35 cm focal length mirror spectrometer, a 7 cm rock-salt prism, and a Nichols radiometer. Except for certain improvements, it is fully described elsewhere[2] and need not be mentioned here. A considerable portion of the work was repeated to 7.5 μ, using mirrors of 1 m focal length and 20 cm aperture, mounted on a large spectrometer.

With this large apparatus the spectrometer slits were 2′ of arc, while in the smaller they were 4′ of arc on the spectrometer circle, so that for the larger apparatus the dispersion was comparable to that of fluorite. With it numerous bands were resolved from 6 to 7 μ, but only occasionally were small bands found in the transparent region, already mentioned, from 4 to 5 μ, while the 3 to 3.5 μ region was sometimes found complex.

For the gases a glass cell 5.7 cm long was mounted in vertical ways, between the spectrometer slit and the incident energy, which was supplied by the heater of a Nernst lamp.

For the liquids different kinds of absorption cells were used. Those having boiling-points below 100° C. were placed in rock-salt cells, made by bending a fine wire, 0.08 to 0.16 mm in thickness, into a U-shape, covering it with Le Page's glue, and placing it between two plates of rock-salt. After drying, the glue was not attacked by the liquids examined. The top of the cell was covered with tinfoil. The plates of rock-salt were split from the natural crystal, about 2×3 cm on an edge, and were more satisfactory than those polished by hand.

[1] COBLENTZ, *Phys. Rev.*, **17**, 51, 1903. [2] *Ibid.*, **16**, 35, 1903.

Liquids boiling above 100° C. could be used in thinner films, which was an advantage on account of their opacity. For these a ring of tin-foil 0.01 mm in thickness was placed between the plates of rock-salt. Around the outside edge of the plates was placed a strip of pure tin, which was 0.1 mm thick, and hence easily bent to fit the cell, thus preventing evaporation. This form of cell is much better than that used in previous investigations, in that it can be thoroughly cleaned, while a new tin-foil ring was used for each new compound.

A block of wood having an opening cut in it over which the rock-salt cell was securely mounted, was placed in vertical ways before the spectrometer slit. The radiation from the Nernst heater passed through the opening in the block and through the rock-salt cell into the spectrometer slit. A clear plate of rock-salt was mounted directly below this cell. In this manner no radiation except that which passed through the cell or clear piece of rock-salt could enter the spectrometer.

The method of observation consisted in projecting successive portions of the spectrum upon the radiometer vane and noting its deflection when the absorption cell was before the collimator slit, and also the deflection when the clear piece of rock-salt was substituted. The ratio of the deflection through the cell to that through the plate of rock-salt gave the transmission through the liquid directly, and more accurately than by finding the absorption of the empty cell, and deducting it. This also meant the reduction of the work by almost one-half. After two months' use the difference in absorption of a plate of the absorption-cell and the "clear plate" was only 3.2 per cent. beyond 3 μ, which is of no significance, since we are not concerned with the question of total absorption.

One of the chief difficulties in this work is to obtain pure chemicals, and it is of the greatest importance to prevent contamination while investigating them. The compounds were imported directly from Kahlbaum, and were the purest obtainable.

In addition to the usual chemical methods of purifying the gases, fractional liquefaction and fractional distillation, in liquid air, was used. By this method they were obtained quite pure, especially ethylene, which showed a purity of 98.8 per cent. The details of all this work will appear in the complete report.

EFFECT OF STRUCTURE.

In order to learn what effect a group of atoms in a molecule has upon infra-red absorption spectra, the most logical procedure is to study isomeric compounds, for the purpose of determining fully that the phenomenon is intramolecular, and after that, to attempt to locate the particular group of atoms suspected of causing the disturbance. As already mentioned, in many cases the spectra of isomeric substances are very similar until we extend our observations far into the infra-red. The examples selected for this paper are representative of all the isomeric bodies studied. The total number studied is so large and varied, while the change in the spectra of pairs of isomers is so marked, that there can be no question that this is due to structure rather than to impurities. In the case of thymol and carvacrol, $C_{10}H_{14}O$, the effect of changing the OH group manifests itself to a marked degree at 5 and 6 μ, while from 9 to 14 μ the spectrum is entirely rearranged.

In aniline, $C_6H_5NH_2$, and its isomer, picoline, $C_5H_4N(CH_3)$, the effect of structure is still more marked. The benzene band at 3.25 μ, found in aniline, is entirely obliterated by the one at 3.35 μ in picoline, while in the spectrum of picoline only one band, at 10 μ, is in common with that of aniline.

In the sulphocyanates, *R-SCN*, and the mustard oils, *R-NCS*, the effect of structure is still more pronounced. As previously mentioned, the small band of the sulphocyanates at 4.68 μ is completely outclassed by the 4.78 μ band in the mustard oils. As the band occurring from 3 to 3.4 μ is a characteristic of carbohydrates, so is this band a characteristic of the mustard oils.

Of all compounds studied, the mustard oils are unique in having an enormous absorption band in the region of short wave-lengths, this side of 5 μ. In carbon disulphide the first strong band occurs at about 6.7 μ, in methyl iodide at 11.35 μ, and in carbon tetrachloride at 13 μ.

In allyl mustard oil, C_3H_5NCS, using the large spectrometer, this band was found to be complex, being opaque from 4.5 μ to 4.9 μ with the maximum located at about 4.8 μ. Phenyl mustard oil, C_6H_5NCS, is still more interesting, since it contains the 3.25 μ band as well as several others belonging to benzene, and has in addition

this strong band of the mustard oils, located at 4.8 μ, just as though the CH and the CS ions were vibrating side by side, but independently of each other.

Other isomers like pinine and limonene, $C_{10}H_{14}$, have a great similarity until we arrive at 10 μ, while the two caproic acids are identical to 6 μ, and begin to show dissimilarity at 8 μ. Probably the most evident example of the influence of structure is in the aliphatic or chain-linked group of atoms, like octane, and the carbocyclic or ring compounds, like benzene. If we consider simply the ***number*** of atoms in the molecule, then benzene, C_6H_6, can be designated by the formula C_nH_{2-6n}, and can be classed with the chain series, C_nH_{2n-2}, C_nH_{2n}, and C_nH_{2+2n}. Hence, reasoning from the fact that, in the three groups of chain compounds studied, *all* the conspicuous bands occur in common, one would expect at least a few of these bands to occur in the benzene, C_nH_{2n-6}, series. But no such coincidence occurs (in Table I, benzene, octane, and tetracosane are shown as typical examples), and only after the substitution of CH_3 groups for H atoms in benzene do we find bands, *e. g.*, 3.43 μ, in common with those of the chain compounds. If, then, we had no knowledge of these compounds, gained from organic chemistry, the evidence presented in the benzene curve and in the curves of octane and tetracosane would be sufficient to show that we are dealing with two distinct classes of compounds.

EFFECT OF MOLECULAR WEIGHT.

In the visible spectrum, Schönn[1], using columns 1.6 to 3.7 m in length of methyl, ethyl, and amyl alcohol found a shifting of the absorption band of methyl alcohol at 0.6430 μ to 0.6515 μ in ethyl, and to 0.6591 μ in amyl alcohol. This was followed by Gerard Krüss,[2] who examined sixty-four different compounds dissolved in CS_2, $CHCl_3$ and C_2H_5OH. He found that by the substitution of a methyl, ethyl, oxymethyl, or carboxyl group in a compound the maximum of the absorption band is shifted toward the red. On the other hand, by substituting a nitro or amido group the absorption band is shifted toward the violet. Subsequent writers on the subject of absorption spectra, in quoting this work, always mention the shift

[1] *Wied. Ann.*, (2) **6**, 267, 1879. [2] *Zeit. f. Phys. Chem.*, **2**, 312, 1888.

toward the red, but rarely mention the shift to the violet. In quoting such a complete investigation which records two well-defined series of phenomena, apparently opposed to each other when considering the question of molecular weight, it seems highly desirable to have the complete observation rather than the part which fits the particular problem under investigation. This is especially desirable in work like that of Ransohoff[1], who thought that a small sharp band found at 4.9 μ in CH_3OH was shifted to 5.2 μ in C_2H_5OH, "which would be an example like that of Krüss." He found no shifting for larger bands.

The following is what Krüss observed for indigo:

TABLE I.

Shift to red	Indigo in $CHCl_3$	$\lambda_{max.}$ at 0.6048 μ
	Methyl indigo in $CHCl_3$	$\lambda_{max.}$ at 0.61917
	Ethyl indigo in $CHCl_3$	$\lambda_{max.}$ at 0.6526
Shift to violet	Indigo in $CHCl_3$	$\lambda_{max.}$ at 0.6048
	Nitro-indigo in $CHCl_3$	$\lambda_{max.}$ at 0.5858

	Water Sol.	Alcohol Sol.
Fluorescin[2]	$\lambda_{max.}$ 0.494 μ	$\lambda_{max.}$ 0.4808 μ
Dibrom	0.5048	0.5094
Tetrabrom	0.5159	0.5251

This is a shift of 0.0055 μ per atom of *Br*, which proportionality was found not to hold true.

In the present work the results agree with that of Krüss, in so far as it seems permissible to assume that the occurrence of a certain conspicuous absorption band in a different place is a real shift. Several of the benzene derivatives are the most noticeable examples (Table I). In benzene, C_6H_6, the maximum occurs at 3.25 μ and is shifted to 3.3 μ in toluene, $C_6H_5CH_3$, to 3.38 μ in the xylenes, $C_6H_4(CH_3)_2$, and to 3.4 μ in mesitylene, $C_6H_3(CH_3)_3$. In other words, by substituting three CH_3 groups for an H atom we have shifted the maximum from 3.25 μ to 3.4 μ. Of all the compounds studied, excepting the gases, this is the only example where such a supposed shifting occurs. For a shift toward the shorter wave-lengths, the amido (amino) derivatives of benzene are the most conspicuous, just as found by Krüss. In aniline, $C_6H_5NH_2$, we find

[1] *Loc. cit.* [2] E. VOGEL, *Wied. Ann.*, **43**, 449, 1891.

the benzene band almost obliterated and the minimum shifted to 2.97μ just as in ammonia, while in picoline, $C_5H_4N(CH_3)$, we have the nitrogen band at 2.92 μ and a second band at 3.35 μ. It is to be noticed that in the xylenes $C_6H_4(CH_3)_2$ and in pyridine, C_5H_5N, the benzene band at 3.25 μ has not been entirely obliterated, just as though there were different resonating ions, benzene, NH_2, and CH_3, vibrating side by side. This is more evident in xylidine and the mustard oils.

In xylidine, $C_6H_3(CH_3)_2NH_2$, which has an NH_2, and two CH_3 groups, we have the representative bands found in ammonia, at 2.95 μ, and in compounds predominating in CH_3 groups at 3.43 μ. The structural formula of aniline indicates that in the original benzene ring an H atom has been replaced by an NH_2 group, while in picoline we have a double benzene ring, containing an N atom and a CH_3 group. The absorption spectra support this theory, for in the aniline spectrum we have the original benzene band at 3.25 μ and the NH_2 band found in ammonia, xylidine, etc., while in picoline we have the benzene band obliterated and the CH_3 band is substituted. The picoline band occurs at 3.35 μ, the mean of 3.25 μ and 3.43 μ instead of 3.43 μ. Can we say, then, that there is a *real* shifting of the 3.25 μ band of benzene in the xylenes?

It must be remembered that we are integrating through a complex band which with ordinary dispersion cannot be resolved with a bolometer or a radiometer. Hence, when we find the maximum of benzene shifted to 3.3 μ in anisol and to 3.4 μ in mesitylene, and find the original benzene band in aniline, benzaldehyde, etc., it is a difficult matter to decide whether we have a true shifting, or whether we have simply determined the center of gravity of the several unresolved bands. An excellent example of this type is thymol, which melts at 44°. The solid film gave a deep band at 3.2 μ. In the melted condition the film was more homogeneous, and two bands were found at, 2.92 μ and 3.43 μ respectively, instead of the mean at 3.2 μ. Other examples like this have been observed when the layer of liquid under examination was too thick.

Now, this occurrence of a new band beside the old one is just what Krüss observed for his solutions, except that the old band has disappeared and the new one makes it appear as though there was a

shifting of the maximum. It is well known that the eye is insensible to slight variations in intensity in the visible spectrum, and it may be that by his method of dilution the weaker band has disappeared.

In view of the fact that we have such a striking similarity between the phenomena recorded here and those observed by Krüss, it appears highly desirable to make a spectro-radiometric study of dilute solutions, say of indigo, and of methyl and nitro-indigo, in chloroform, to learn whether there is but one band or whether there are two, viz., the original one due to the indigo ion which disappears on dilution, and a second due to the methyl or nitro group of ions, just as in the present work on aniline we have the original benzene band, at 3.25 μ, and a second at 2.97 μ. As already mentioned, it has not yet been shown that the selective absorption of a solid in solution and the intramolecular absorption of the solvent are identical, but the question can be more fully settled by a study of the solutions, as just indicated.

There are other bands farther out in the infra-red which shift back and forth, just as noted above, but the original benzene bands are more numerous and not so well defined, so that it is difficult to discuss them. The most noticeable ones are those of the methyl sulphocyanate at 7.06 μ and 7.61 μ, which occur at 6.91 μ and 7.27 μ in ethylsulphocyanate.

However, in all the benzene derivatives studied, the occurrence of an apparently new band in the derivative does not always seem to be *new*, but simply that the derivative has brought about a condition within the molecule such that the original resonating ion has greater freedom.

In gases there is a more definite shifting of the absorption band lying between 3 and 3.5 μ, as shown in Table II:

TABLE II.

Gases	Maxima	Maxima
Acetylene, C_2H_2	3.08 μ	7.38 μ
Ethylene, C_2H_4	3.28	6.98
Ethane, C_2H_6	3.39	6.85
Butane, C_4H_{10}	3.42	6.85
Methyl ether, $(CH_3)_2O$	3.45	6.88
Ethyl ether, $(C_2H_5)_2O$	3.45	7.00
Methane, CH_4	3.31	7.70

In the region of 6.8 to 7 μ there is a somewhat similar shifting, but there is less regularity in the positions of the bands. Ångström[1] has shown that the occurrence of the CO_2 band at 4.28 μ and of the CO band at 4.59 μ invalidates the assumption that the position of an absorption band depends upon molecular weight.

Ransohoff's[2] work on the alcohols shows that for the alcohols there is no shifting with increase in molecular weight.

Within the limits of experimental error, Puccianti's[3] work for the region of 1.71 μ shows no shifting of the maximum of an absorption band.

In all my work on the different compounds like methyl, and ethyl iodide, nitrate, cyanide, aniline, etc., no shifting can be detected. To make this test conclusive for the marked band at 3.43 μ, this region was repeated for both compounds (*e. g.*, methyl and ethyl iodide), before setting the spectrometer for another part of the spectrum. In this manner a slight shift, noticed in methyl and ethyl iodide, which had been examined on different dates, several months intervening, was found not to exist, showing an instrumental error. This method of testing a series of compounds at one region of the spectrum on the same day is the only way to be certain of slight differences of wave-lengths.

Through the generosity of Professor C. F. Mabery, of the Case School of Applied Science, who presented me with twenty-five very pure distillates of petroleum, belonging to the series C_nH_{2n-2}, C_nH_{2n}, and C_nH_{2n+2}, a final test was applied to this perplexing question. The absorption spectra of two of these, octane, C_8H_{18}, boiling-point 118–120°, and tetracosane, $C_{24}H_{50}$, solid, boiling point 274–276° (50 mm), are given in Table I.

In these as well as in all the intermediate ones no shifting could be detected, although the greatest efforts were made to do so. This was not a little surprising, for according to the measurements on the alcohols by Schönn (*loc. cit.*), in the visible spectrum, a shift for this greater number of CH_2 groups should have occurred. Even if we assume that the shifting is least for the infra-red and increases as we approach the ultra-violet, unless the total shift for this increase of 16 CH_2 groups [Octane, $C_8H_{18}=CH_3(CH_2)_6CH_3$, tetracosane, $C_{24}H_{50}$

[1] *Öfversigt Kongl. Vet. Akad.*, No. 7, 331, 1890. [2] *Loc. cit.* [3] *Loc. cit.*

$=CH_3(CH_2)_{22}CH_3$] is less than 0.03 μ, it is safe to assume that no shifting occurs. A shift of 0.03 μ at 3.5 μ is 20″ of arc on the spectrometer circle, and at 7 μ it is 1′ of arc, for the small spectrometer, while on the larger apparatus the values in divisions of the spectrometer circle are *twice as great*, viz., 40″ and 2′ of arc, so that it would have been impossible to escape detection, especially such sharp, well defined bands like the one at 3.43 μ and that at 6.86 μ.

An interesting fact to be noticed in this connection is that *all* the prominent lines found in the two oils just mentioned, are present in *all* the petroleum oils studied, as well as in many other compounds like myricyl alcohol, piperidine, etc. For a larger dispersion the transparent region at 4 to 6 μ remains so for some oils while in others numerous small bands were found.

The difference between the spectra of the oils (aliphatic series) and the benzene spectrum (carbocyclic series) has been noticed under the question of structure. The benzene spectrum as well as that of its methyl derivatives is banded, "channeled," *i. e.*, the lines occur in groups, just as Pauer[1] found for the ultra-violet. He found the bands of the benzene spectrum, which extend from 0.235 μ to 0.260 μ, condensed and shifted toward the visible spectrum for toluene, the xylenes, aniline, etc., and considers it due to increase in molecular weight. If we consider the center of gravity of the benzene bands at about 0.245 μ, and that of the methyl derivatives, *e. g.*, toluene at about 0.267 μ, this shift amounts to 0.02 μ, while for aniline it is about 0.05 μ.

THE EFFECT OF CERTAIN CHARACTERISTIC GROUPS OF ATOMS.

Having found that infra-red absorption spectra depend upon the internal structure of the molecule, and that their maxima are not influenced by molecular weight, the next step is to determine, if possible, what groups of atoms, or "ions," have the power of absorbing heat-waves. This is of considerable importance, since many recorded phenomena have been credited to the "resonance of the *OH* ion in the molecule." Aschkinass (*loc. cit.*) found the absorption bands of water at the wave-lengths 1.51 μ, 3.06 μ, and 6.1 μ. Although he says little about the sequence of the maxima, subsequent writers

[1] *Ann. der Phys.*, **61**, 363, 1897.

have laid great stress upon this harmonic relation, showing electromagnetic resonance. Marx[1], in measuring the dielectric constants of water for electrical waves, finds a double harmonic relation for the electrical region. Ransohoff (*loc. cit.*) found the alcohol bands harmonic at 1.71 μ, and 3.43 μ. Although the alcohols were "chemically pure," that is a different question from the one of having them "water-free," which he does not consider, and the bands found by him at 3 μ and 6 μ may be due to water. The higher alcohols like glycerin, even if they could be freed from water, are so hydroscopic that they are difficult to investigate.

In the present work alcohols were avoided because of their great opacity beyond 7 μ, as well as on account of the difficulty in freeing them from water. Only one alcohol was studied, viz., myricyl, $C_{30}H_{60}OH$, which is a solid obtained from beeswax. A very thin solid film was obtained by melting between two plates of rock-salt. The spectrum is very marked for several strong absorption bands which correspond to those in the petroleum distillates. The dispersion with the large spectrometer at 3 μ is comparable with that of fluorite, and hence comparison can be made with the work of Ransohoff, who found the maxima of the alcohols at 1.71 μ, 3 μ, 3.43 μ, and 6.06 μ. In the present work the maxima occur at 1.71 μ, 2.95 μ, 3.43 μ, and 5.8 μ. The water bands were found at 2.95 μ and 6 μ, so that the 2.95 μ band coincides with the one for water, while the one at 5.8 μ does not. The disagreement with Ransohoff at 3 μ may be due to inaccuracies in the dispersion curve of rock-salt, which passes through a double curvature at this point, and hence is difficult to determine.

Two explorations were made, the first with the myricyl alcohol just taken from the containing bottle. For the second examination several grams of the myricyl were heated to 110° for seven hours, in a drying oven, which was sufficient to expel any water present. Immediately after the heating, a thin solid film was examined, and the bands at 2.95 μ and 3.43 μ coincided exactly with those found previously, which would indicate that the 2.95 μ band is not due to

[1] MARX, "Potential und Dissociation in Flammengasen," *Ann. der Phys.*, **2**, 795, 1900 (also on Electromagnetic Resonance, *Wied. Ann.*, **66**, 600, 1898), after a lengthy discussion concludes that, although it is a plausible assumption, it has not been proved that electrolytic dissociation in a flame depends upon the effect of the electromagnetic resonance of the *OH* ion upon infra-red radiation.

water. Whether it is due to the *OH* group is a different question. If it is due to *OH*, then the 5.8 μ band should coincide with the water band found at 6 μ.

Numerous other compounds having *OH* groups were studied. In Table III are given a series of compounds which have absorption bands at about 3 μ.

TABLE III.

Compounds	Maxima	Remarks
Water, HOH	2.95 μ	Depth is 70 per cent. as found with larger spectrometer
Thymol, Carvacrol $C_{10}H_{13}OH$	2.92	
Eugenol, $C_{10}H_{11}O\text{-}OH$	2.89	
Methyl salicylate, $CH_3OOC\text{-}C_6H_4OH$	3.1	Depth 70 per cent., probably the mean of the 3.25 and 2.95 μ bands.
Menthol, $C_{10}H_{19}OH$	3.0	50 per cent., comparison spectrum of H_2O at 2.95
Phenol, C_6H_5OH	2.97	60 per cent.
Ammonia, NH_3	2.92	30 per cent.
Pyridine, C_5H_5N	2.95	30 per cent.
Picoline, $C_5H_4N(CH_3)$	2.92	Band shallow, 3 per cent.
Piperidine, $C_5H_{11}N$	3.00	30 per cent.
Aniline, $C_6H_5NH_2$	2.97	70 per cent., very sharp
Xylidine, $C_6H_3(CH_3)_2NH_2$	2.95	50 per cent.
Pyrrol, $C_4H_4(NH)$	2.95	70 per cent.
Eucalyptol $C_{10}H_{18}O$	2.90	30 per cent. oxide, does not contain an *OH* group
Terpineol $C_{10}H_{18}O$	2.93	30 per cent.

In Table III it will be noticed that ammonia also has a band near that of water, and at a slightly less wave-length. Considerable time was spent in showing that it was not due to water-vapor. The gas was fractionally liquefied and distilled, and then placed in a glass pipette containing freshly heated calcium oxide, over mercury, for eight days. At the end of this time the absorption band coincided exactly with the one previously found, showing that the band is characteristic of ammonia. Furthermore, it will be noticed that the compounds containing the amido, NH_2, group, and certain ones containing nitrogen, have a characteristic band in this region. These compounds were dried with potassium carbonate which would have removed traces of water. Other compounds like the aldehydes and

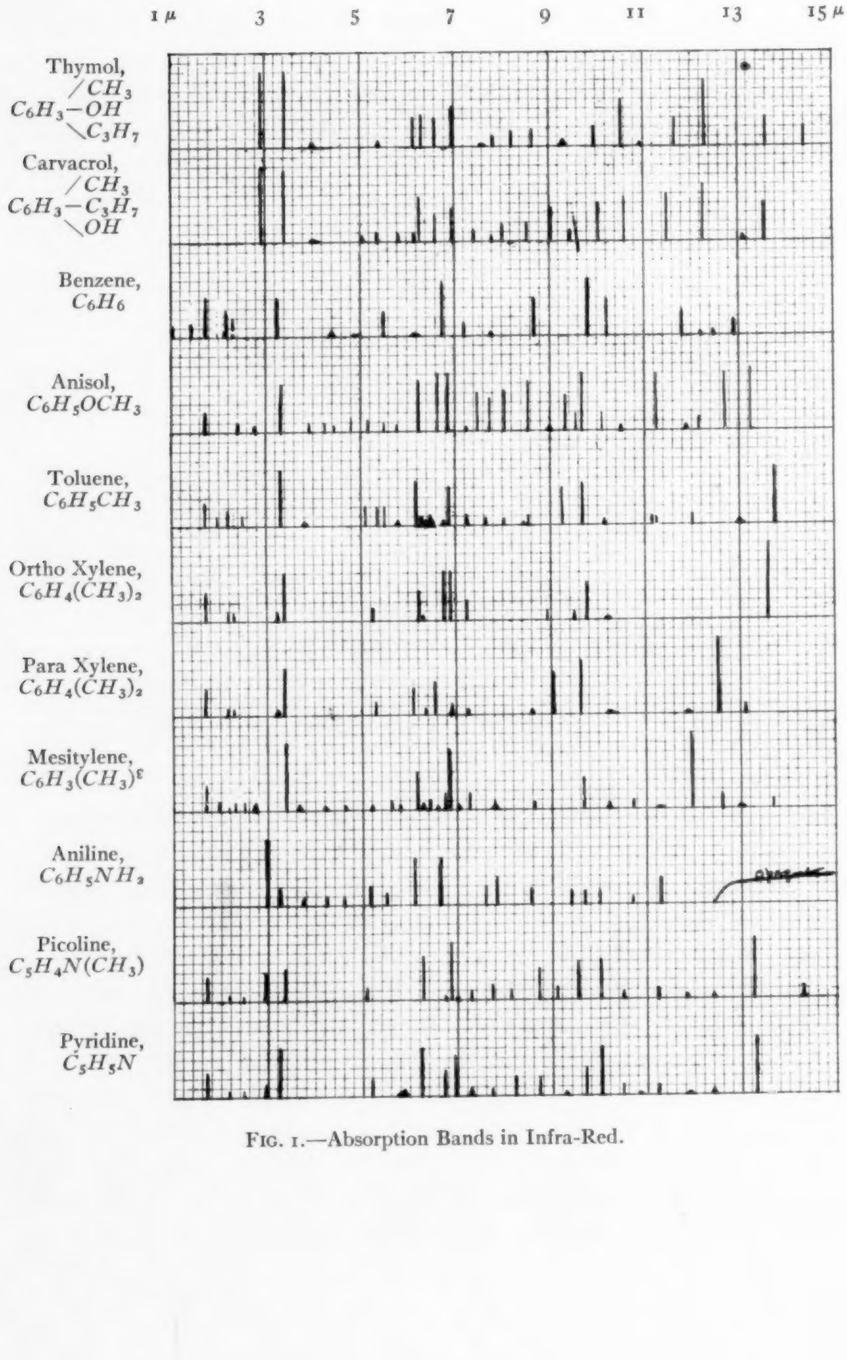


FIG. 1.—Absorption Bands in Infra-Red.

the fatty acids *do not* show this band. Commercial ethyl ether contains about 3 per cent. of water, but there is only a slight depression in the absorption curve at 2.95 μ. The fatty acids, *e. g.*, caproic, stearic, etc., are of interest because they have no band at 2.95 μ. In electrolysis, the alcohols are separated into ethyl and OH ions, while in the fatty acids, instead of the OH ion we have an H ion. Hence, reasoning from this analogy, one would not expect a band at 2.95 μ for the fatty acids. In the other compounds having an OH group, *e. g.*, eugenol, thymol, menthol, and phenol, strong bands are to be found, shifting from 2.89 μ to 3 μ. They show *no* bands at 6 μ. Can we assume, then, that the bands at 2.9 to 3 μ are due to OH? At the present writing the evidence is not very favorable. Considering the band of ammonia at 2.92 μ and those of compounds containing NH_2 or nitrogen, the coincidence appears to be somewhat accidental. Further in the infra-red we have numerous cases of the coincidence of absorption bands.

As a whole, the most definite conclusion we can draw at present is that the alcohols have a characteristic band at about 2.95 μ just as the band at 4.78 μ is characteristic of the mustard oils.

In myricyl alcohol the bands at 1.71 3.43, 6.86, 10.2, and 13.88 μ are closely harmonic. Puccianti (*loc. cit.*) observed that the band at 1.71 μ occurs in *all* compounds which have a C joined directly to an H atom. This has been verified in the present work, on seventeen new compounds, and it appears that this is a general case. But in benzene the next maximum occurs at 3.25 μ, and the following at 6.75 μ, which would indicate that the harmonic relation in other compounds is somewhat accidental. There are however, numerous pairs of lines, especially in ammonia, which from the "constant differences" of their vibration numbers indicate spectral series. A large band at 5.8 μ is of frequent occurrence in numerous benzene derivatives, as well as in the aliphatic compounds. The same is true of the 6.8 μ band found in all the petroleum oils, in a few benzene derivatives, and in piperidine, which consists of a ring of CH_2 groups. It thus appears that we have a certain vibrating ion which is present in numerous compounds.

The CH_3 group of atoms is probably the most important to be considered, but only a few cases can be mentioned here. The most

noticeable effect is in benzene derivatives. It was shown under the discussion of the effect of structure that the benzene group, C_6H_6, although it appears as a series, C_nH_{2n-6}, is entirely different from the chain compounds like C_nH_{2n-2}, etc. But a substitution of several

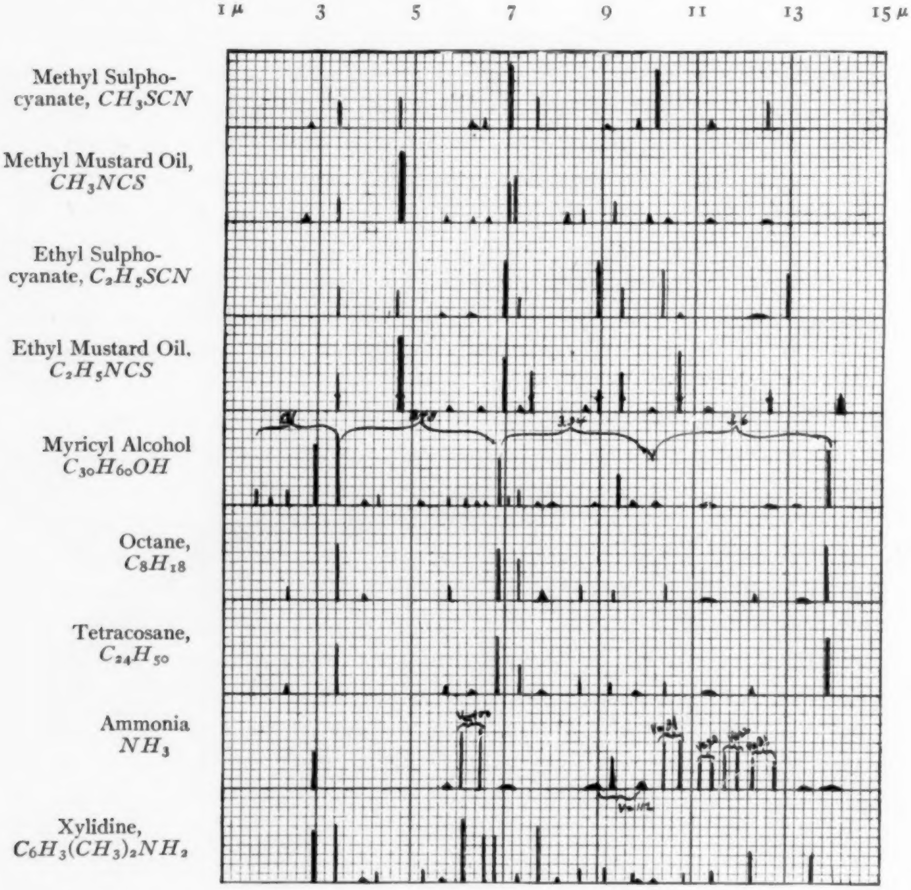


FIG. 2.—Absorption Bands in Infra-Red.

CH_3 groups completely absorbs the 3.25 μ (benzene) band, and the 3.43 μ band which is characteristic of all compounds containing CH_3 groups takes its place. Whether the 3.25 μ band has actually disappeared is an open question. In mesitylene there is still a trace of the 6.75 μ band of benzene, showing that the effect of the benzene

ion has not been destroyed by the substitution of three CH_3 groups. In the xylenes the 6.75 μ band is least affected, while the 3.25 μ suffers the most, and the whole strengthens the belief, mentioned in the beginning, that certain vibrating ions always seem to be present, but that their effect in absorbing heat-waves seems to depend upon the damping effect of surrounding groups of atoms. The effect of substituting an NH_2 group for an H atom, thus forming aniline, has the least effect on the benzene, 3.25 μ band, while those from 6 to 7 μ have entirely disappeared. In benzaldehyde, C_6H_5CHO, the 3.25 μ band is not seriously influenced by a more intense absorption maximum at 3.55 μ, while in benzonitrile, C_6H_5CN, and in C_6H_5Br the 3.25 μ band suffers no change.

Are we then to conclude that a certain group of atoms, which behaves in a certain definite manner in chemical reactions, absorbs heat-waves in the manner just noted; or are we to consider the effect due to the bonding of the atoms in the molecule; or is the effect due to both causes?

Only after a thorough study of the data at hand will it be possible to attempt a reply to this, as well as to numerous other questions not considered here.

Physical Laboratory,
Cornell University,
June 1, 1904.

MINOR CONTRIBUTIONS AND NOTES.

ON THE PHYSICAL NATURE OF THE SOLAR CORONA.[1]

In a paper on "The 1900 Solar Eclipse"[2] Langley and Abbot published some measurements on the heat radiation of the solar corona. This was found to be "unexpectedly feeble." Therefore the authors oppose the view that the "main source of the light from the corona is the incandescence of its particles, due to the proximity of the hot photosphere." Instead of this they suppose that the "principal source of its radiations is of the nature of an electrical discharge." The example used by the authors for such light caused by electrical discharge, namely, the light of the aurora, seems to me not to be a good one, for the aurora has a bright-line spectrum and the corona mainly a continuous one. Again, the phosphorescent glow under the influence of cathode or Röntgen—or ultra-violet—rays seems to give a nearer approximation to the continuous spectrum of the corona. But the spectra of the coronas of 1898 and 1901, as given by Campbell and Perrine, are not very similar to that of a phosphorescent substance, so that this modification of Langley and Abbot's views has also to struggle with rather great difficulties.

Mr. Campbell drew my attention to the conclusion of Langley and Abbot. As will be seen from the following considerations, the heat—and light—effects of the corona agree very well with the assumption that its radiation is due to the incandescence of its particles. The special difficulty of Langley and Abbot is given in the following words: "Why should the coronal radiation, of equal apparent brightness to that of the full Moon, be comparatively so feeble in heating effect?" This effect was measured by five scale-divisions on a bolometer, whereas a (black) body of the room's temperature under similar conditions gave eighteen divisions.

Now I shall show that the brightness and the heat-effect of the corona are just such as we might expect from dust-particles heated by the radiation from the photosphere of the Sun.

The image of the Sun was produced by a concave mirror of 1 m focal length. It had therefore a diameter of 9 mm. The part of the corona observed was 0.2 mm from the limb of the Sun. I assume that this means the edge of the photosphere. From this I calculate that the photosphere

[1] *Lick Observatory Bulletin* No. 58. [2] Washington, 1904, p. 26.

of the Sun, seen from the observed point of the corona, would fill an angle of sight equal to $2^\circ.73$, corresponding to a spherical angle equal to $2\pi \cdot 0.708$. A spherical particle in the corona at this point radiates to space through an angle of $1.292 \cdot 2\pi$, and to the Sun through an angle of $0.708 \cdot 2\pi$. Equilibrium of heat is soon reached and gives, in accordance with Stefan's law, if t is the absolute temperature of the particle, T that of the Sun (6000° absolute),

$$1.292\, t^4 + 0.708\,(t^4 - T^4) = 0;$$

from which $t^4 = 0.354\, T^4$, and as T is assumed to be 6000°, t is found to be 4620° absolute.

According to the law of Wien (or Planck) we may now calculate the relative strength of the radiations E_1 and E_2 of two bodies at the temperatures 6000° and 4620° for different wave-lengths (λ). Thus we find:

$\lambda = 0.1$	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$E_1 = 0.0041$	2.0	142	248	270	245	202	162
$E_2 = 0.0000033$	0.052	13.2	41.7	64.9	74.7	73.0	66.1

Taking the sum of the radiations in the visible spectrum from $\lambda = 0.4$ to $\lambda = 0.8$, we find $\Sigma E_2 : \Sigma E_1 = 0.289$. If we only regard the physiologically most effective part in the spectrum, for $\lambda = 0.55$, we find $\Sigma E_2 : \Sigma E_1 = 0.271$, which falls very close to the first figure. We may use the mean of them, 0.28, as lying very near to the true figure.

A body of the temperature 4620° abs. would therefore radiate a quantity of light equal to 0.28 times that radiated from the Sun, if they took up equal spherical angles of sight. The same would be the case if the body of 4620° were composed of little particles in such numbers that the background (the sky) could not be seen between them. Now, instead of this, Langley and Abbot indicate that the brightness of the corona was equal only to that of the full Moon, which in accordance with the measurements of Zöllner, is supposed to be about 1:618000 of that of the Sun. Therefore we must conclude that only (1:618000):(0.28) = 1:173000 of the angle of sight to the corona is filled by the radiating particles of the corona, in the case of Langley and Abbot's observations.

In this calculation we have neglected the sunlight reflected from the particles in the corona. If we had not done so, we should have found a number somewhat, but not much, less than 1:173000.

Let us now make an analogous calculation for the radiation of heat from the corona. Langley and Abbot compare this with the radiation of heat from the Moon. Langley found[1] that the radiation of the Moon in the middle of the Earth's umbra is equal to 1.3, whereas the radiation from the

[1] *Memoirs Nat. Acad. of Sciences*, **4**, Part II, p. 159.

middle of the full Moon is equal to 180 scale divisions. Now the middle of the full Moon has a temperature of about 419° abs. Calculating the temperature of the middle of the umbra according to Stefan's law, we find 122° abs. As the temperature of the Moon sinks from 419° abs. to 122° abs. in about three hours, it will be quite near to the truth to suppose that at the solar eclipse the center of the Moon will be very near to absolute zero, so that its radiation may be neglected. Now we will suppose that the temperature of the cardboard, used for screening the bolometer in the experiment, was +27° C. = 300° abs., which will not be far from the truth. Further, we will suppose with Very[1] that the sky acts as a screen, sending back 50 per cent. of the radiation from the bolometer. Then the radiation from the bolometer to free space being —y*, it is to space with the intervening atmosphere of the Earth, $-\frac{1}{2}y$. This radiation gives a deviation of −18 divisions.

Now the bolometer was directed to the corona and the deviation found to be −13 divisions. The loss of heat to space through the atmosphere was as before $-y/2$. To this came the radiation from the corona, which is supposed to be absorbed by the atmosphere (or reflected by its dust-particles) to a degree of 50 per cent. Call it therefore $z/2$, so that we have

$$-y/2+z/2=-13$$

or $z=10$. Now we have $y=36$ as a measure of the radiation of a black body of 300° abs. If we suppose, as before, that the particles of the corona radiate as a black body, the effective temperature of the corona is found to be $\sqrt[4]{\frac{10}{36}}\cdot 300 = 217^{\circ}.75$ abs.

We have above deduced the temperature of the particles in the corona to be 4620° abs.; that is, the radiation should be $\left(\frac{4620}{217.75}\right)^4 = 2.02\cdot 10^5$ stronger than it is found, provided that space would not be seen through the corona. Therefore we conclude that the particles of the corona do not fill more than the 202000th part of the angle of sight. This figure agrees excellently with that, 173000, calculated for the light, as evidently the measurements of Langley and Abbot are not claimed to give more than the order of magnitude. Also, the neglect of the sunlight reflected from the corona and the deviation of the small coronal particles from the laws of radiation valid for absolutely black bodies would well account for the discrepancy, if the measurements were absolutely correct.

[1] Astrophysical Journal, **8**, 265, 1898.

* The sign − indicates loss of heat from the bolometer.

The difficulty which Langley and Abbot found, as stated above, is discussed in the following quotation (*l. c.*, p. 251).

If the brightness of the full Moon and of the solar corona are to the eye the same, why should they be different in heating effect? Obviously a natural explanation would be the comparative absence of some invisible radiations in the case of the inner corona, which are present in the case of the Moon, such as would be due to the absence of infra-red rays.

We will analyze this point a little more closely. If we regard the radiation of the Sun as like that of a body with a temperature of 6000° abs., and count radiation between $0.4\,\mu$ and $0.8\,\mu$ as luminous, and the rest as non-luminous radiation, the proportion of luminous to the total radiation is as 0.45 : 1, calculated from Wien's law. If we now regard a point in the middle of the full Moon, we find that it reflects diffusely 12 per cent. of the Sun's light falling upon it (Zöllner), while the rest is transformed into heat. For the non-luminous radiation the Moon is supposed to be a nearly opaque body, and it reflects diffusely perhaps 2 per cent. of the incident non-luminous radiation and absorbs the rest. Now, all the absorbed radiation is (as equilibrium of heat is reached) radiated out as heat.° The proportion of the luminous and total radiation from the Moon is therefore in the proportion

$$\frac{0.12 \cdot 0.45}{1} = 0.054 : 1.$$

For the particles of the corona Wien's law gives the proportion of luminous to the total energy as 0.36 : 1, neglecting the sunlight reflected from the corona. We may therefore say that the luminous energy is more than 6.7 times stronger in the coronal light than in the moonlight, compared with the total energy. It is this distribution of the radiation which makes the corona appear so luminous and so "cold" compared with the Moon.

By help of the measurements of Abbot it is possible to form an idea of the mass of the coronal dust. The dust-particles in the inner corona are heated to 4620° abs., and may therefore be drops of liquid metal. It is not probable that the dust is of carbon, for carbon seems to have a rather high vapor-pressure at this temperature, and the gaseous pressure in the corona is extemely low. We may, for example, suppose that the dust-particles are molten iron drops (melting-point of iron about 1600°C.). The specific weight of iron is 7.9 at 0°C., of molten iron 6.9 (at 1600°C.), and if its cubical dilatation is like that of cast-iron (0.000053 between 0° and 1000°, according to Le Chatelier), its specific weight at 4620° abs. will be about 6

° Therefore it is for our calculation immaterial whether the non-luminous radiation is partially reflected or not.

(probably it is lower). If the iron drops reflected the incident light totally, they would be driven away by the radiation-pressure from the Sun with the same force as that with which they would be attracted to the Sun if their diameters were 250 $\mu\mu$. According to Schwarzschild's calculations for the influence of diffraction and the deficiency of the total reflection, this figure may bechanged to about 350 $\mu\mu$.

It is very probable that those drops for which gravitation is just compensated by the pressure of radiation will be the chief material of the inner corona. For drops of other sizes are selected out, the heavier ones by falling back to the Sun, the lighter ones by being driven away by the pressure of radiation, so that just the drops which, so to say, swim under the equal influence of gravitation and pressure of radiation will accumulate in the corona. The mass of such a drop is $m = \frac{4}{3}\pi\,(175)^3 \times 10^{-21} \times 6 = 0.135 \times 10^{-12}$ grams. Their cross-section is $a = \pi\,(175)^2 \cdot 10^{-14}\,\mathrm{cm}^2 = 9.62 \cdot 10^{-10}\,\mathrm{cm}^2$. Now we know that the particles in the corona fill about $\frac{1}{190000}$ of the angle of sight. Therefore a column of 1 m^2 cross-section through the corona at the point of observation of Abbot will contain $\frac{1}{190000} \cdot \frac{10^4}{9.62 \cdot 10^{-10}}$, $= 54.7 \cdot 10^6$ particles of a total mass equal to $7.37 \cdot 10^{-6}$ grams.

According to a formula by Turner, the brightness of the corona decreases in moving from the photosphere, inversely as the sixth power of the distance from the Sun's center. This may be due to a decrease of the temperature, but certainly the quantity of reflecting particles exerts the chief influence, and therefore I have supposed as a first approximation that the number of particles in a volume of 1 cubic meter is inversely proportional to the sixth power of the distance from the Sun. Then, according to Turner's formula, the concentration of matter in the innermost part of the corona would be 1.3 times greater than that calculated above. The total quantity of particles that we observe in the corona would, according to Turner's formula, be half as great as the quantity of particles in 1 m^2 at the innermost part of the corona, multiplied by the cross-section of the Sun in square meters. It would therefore be $\frac{1.3}{2} \cdot 54.7 \cdot 10^6 \cdot (108.56)^2 \cdot \frac{5.1}{4} \cdot 10^{14} = 62.8 \cdot 10^{24}$ particles with the total mass $8.6 \cdot 10^{12}$ grams.[1]

Now, the mass of the corona will be a little greater than this figure shows, for what we see is the total corona diminished by the part of it lying before and behind the Sun. I have executed a mechanical integration of the form-

[1] The surface of the earth is $5.1 \cdot 10^{14}$ m^2 and the radius of the Sun 108.56 times greater than the Earth's radius.

ula for this case, and found that the total mass of the corona is $\frac{667}{390}$ times greater than of that part calculated above, that is $14.7 \cdot 10^{12}$ grams. If we had supposed that another material than iron composes the corona, we should have found nearly the same total weight; so for instance, if the specific weight had been one, the total weight would have been about $11 \cdot 10^{12}$ grams. If the diffraction did not disturb, and the reflecting and absorbing power were always the same, then the calculated weights would be independent of the specific weight of the material of which the drops consist. For the total of the cross-sections of the drops, determined by the radiation, is an experimental constant, and the diameter of the drops would under these conditions be inversely proportional to their density. Now, the total mass is proportional to (total cross-section) $\times$ (diameter) $\times$ (specific weight) and as the first value is constant, and likewise the product of the last two, then the total product, *i. e.*, the calculated mass, is independent of the assumed specific weight. For different metals that may be supposed to enter into the corona the specific weight is also but little variable, and therefore also the condition for diffraction, according to Schwarzschild, is nearly the same for them all. Likewise their reflecting and absorbing power cannot be very different, so that the calculation given above may be regarded as very reliable concerning the order of magnitude.

As now the brightness of the corona seems to vary between the single and the double brightness of the full Moon, the total weight of it may vary in the same proportions, or approximately between 13 and $26 \cdot 10^{12}$ grams.

It is also easy to make a calculation of the probable number of drops in a cubic meter at the densest part of the corona. The number of drops in a cross-section of 1 square meter was found to be $54.7 \cdot 10^6$. The number of particles in a column of 1 square meter cross-section, and the Sun's diameter as height, would be 1.82 times greater (found by mechanical integration), if the number of particles per cubic meter were the same as in the innermost part of the corona. The total number of particles would be then $1.3 \cdot 54.7 \cdot 10^6 \cdot 1.82 = 130 \cdot 10^6$ in $108.56 \cdot 12740 \cdot 10^3 = 138 \cdot 10^7$ m^3, or every particle would take up a space of 10.7 cubic meters. This space would increase proportionately to the sixth power of the distance from the Sun's center, and in a distance of one solar radius would therefore be $64 \cdot 10.7 = 685$ cubic meters. If we had calculated with a substance having n times greater specific weight than the iron, we should have found a volume about n^2 times less, so for particles of the specific weight 12, the volume would have been only 2.7 cubic meters for every particle in the innermost part of the corona.

As a comparison with the corona we may use a dense fog. Conrad determined the quantity of water in a fog, in which one could see 26 steps (equal to about 20 m) to be 4.4 grams per cubic meter. For water drops of 20 μ diameter (probable value) 1 cubic meter of this contains $1050 \cdot 10^6$ drops, with a total cross-section of 0.33 m^2. According to this it is easy to calculate that a thin layer of 0.015 millimeter of this fog would look just as bright as the corona, if the drops consisted of molten iron. Through a wall of such a fog 20 meters thick only the 630th part of an incident light-ray would pass unreflected and unrefracted, which agrees well with the assertion that it is impossible to discriminate objects at 20 meters distance through such a fog.

It is often supposed that the outermost layers of the Sun are of an exceedingly low temperature, due to the adiabatic dilatation of the Sun's gases from their vertical circulation. Just in the same manner we may calculate that the highest strata of the Earth's atmosphere should have an exceedingly low temperature.

The spectroscopic evidence for the Sun gives a totally different idea of the temperature in its upper strata. This depends upon two circumstances. The radiation of the Sun is extraordinarily strong. In the higher strata the density and consequently the heat-capacity of the gases sink to the lowest limit. Therefore their expansion, with the lowering of the temperature in ascending, is wholly overwhelmed by the strong radiation, and we may calculate the temperature as determined by the radiation alone, as we have done above, without committing any sensible error.

This probably also holds good for the uppermost extremely thin strata of the Earth's atmosphere, especially on the insolated side of the Earth. These highest strata contain particles of cosmical dust, supposed to swim by help of the repulsion of their negative electric charges from the electric charge of lower strata. On account of the insolation the temperature of these dust-particles reaches about 57° C., if the temperature of the soil below is about 30° C.,[1] as is easily calculated by the formula of Stefan. Also, on the night side of the Earth, by the radiation of the Earth, these particles will get a temperature $\sqrt[4]{2}$ times lower than that of the soil. If this is assumed to be 15° C. one finds for the dust-particles in the highest strata −31° C. Now much lower temperatures have been observed in lower strata up to about 20 km. It is therefore probable that our atmosphere at a certain height reaches a minimum of temperature, and that at higher strata the temperature again increases. Especially is this valid for the insolated part of the Earth, on which the highest temperatures according to this

[1] Above a soil of 0° C. it would be 47° C.

opinion occur in the highest strata of the atmosphere and not, as is generally supposed, in the lowest layers of it.

These conclusions are in excellent agreement with the results of the most modern researches, by Teisserenc de Bort and Assmann, of the temperature of the highest investigated strata of the air.

SVANTE ARRHENIUS.

LICK OBSERVATORY, MOUNT HAMILTON,
August 1, 1904.

THE RADIAL VELOCITIES OF *S SAGITTAE* AND *Y SAGITTARII*.[1]

MEASURES of seven plates of *S Sagittae*, employing the low-power, sky-standard table published elsewhere in this number, have resulted in the detection of a wide range in the radial velocity of this star. As a whole, these plates are much below the average excellence, but are sufficiently reliable to establish the binary character of this variable. The accompanying table contains the number of the plate, the Greenwich date, the interval since maximum, the velocity referred to the Sun, the number of lines used for each plate, and the temperature-change during the exposure. If the velocities are plotted in the usual way, assuming the identity of the light- and velocity-periods, they are seen to follow a curve in every way similar to those of *η Aquilae* and *W Sagittarii*, and the elements will approximate closely to those determined for *W Sagittarii* above. There is also some evidence pointing to a composite character for the curve.

Plate No.	Date	Interval since Max.	V	Number of Lines	Temperature-Range	Remarks
28 F	1903 Aug. 9.9	$1^{d}4$	$-20^{km}.3$	20	0°.4C.	Underexposed
33 E	14.9	6.4	−32.2	44	0.1	
35 D	16.9	0.0	−30.2	45		
42 B	26.9	1.6	−20.2	20	0.3	Underexposed
54 B	Sept. 6.9	4.2	+3.9	41	0.1	Comparison poor on one side.
72 B	13.8	2.8	−17.3	37	0.0	
3350 C	1904 July 20.7	3.7	−3.9	9	0.0	Underexposed

As the result of relative measures of four lines on two plates, Dr. Stebbins found a range of 9 km in the radial velocity of *Y Sagittarii*. My own measures of nine plates of this star indicate that it is approaching our system, but not at a constant rate. The range of velocity so far observed amounts to 17 km. The form of the curve seems quite different from that of other

[1] Also to appear in a *Bulletin* of the Lick Observatory.

Cepheid variables, but the character and number of the plates available are not such as to warrant any definite conclusions on that point.

RALPH H. CURTISS.

LICK OBSERVATORY,
July 1904.

ON THE SPECTRA OF *R SCUTI* AND *W CYGNI*.[1]

OBSERVATIONS of the spectra of *R Scuti* and *W Cygni* are particularly valuable, as these stars occupy a unique position between the short-period variables on the one hand, and the *o Ceti* variables on the other. Visual observations of *R Scuti* by Espin have led him to suspect the presence of bright lines in its spectrum, but he seems to have been unable to identify them. As far as I know, *W Cygni* had not been observed with the spectroscope.

Exposures upon both these stars with Spectrograph I were begun in the middle of July 1903. They were continued until November 11, 1903, in case of *R Scuti*, and December 28, 1903, in case of *W Cygni*. *R Scuti* was examined visually with the spectrograph until December 7. In the meantime, *R Scuti* rose to maxima toward the end of July and the beginning of October, while maxima of *W Cygni* occurred early in August and in the middle of December. During this period about twenty-five spectrograms of *R Scuti* and twenty of *W Cygni* were secured with Spectrograph I.

On plate 13 E of *R Scuti* (see Plate XIV, 1), H_β, H_γ, and H_δ shone out strongly as bright lines, but they faded quickly until at minimum the spectrum of the star departed but little from the solar type with dark hydrogen lines of customary intensity. At the two subsequent maxima the bright hydrogen lines were not seen, but the intensity of the absorption line at H_γ seemed to have decreased very much as the star approached its maximum of October (see Plate XIV, 2). Judging from rough measures of a few plates, the radial velocity of this star appears to be constant and about +42 km in actual value.

W Cygni shows a banded spectrum of a type characteristic of long-period variables. At the maximum of August 1903 the hydrogen lines mentioned above appeared as strong bright lines (see Plate XIV, 3) which faded gradually to the star's minimum, while the absorption line at *g* broadened greatly during the same period. Other suspected bright lines in the spectrum of this star remain to be identified.

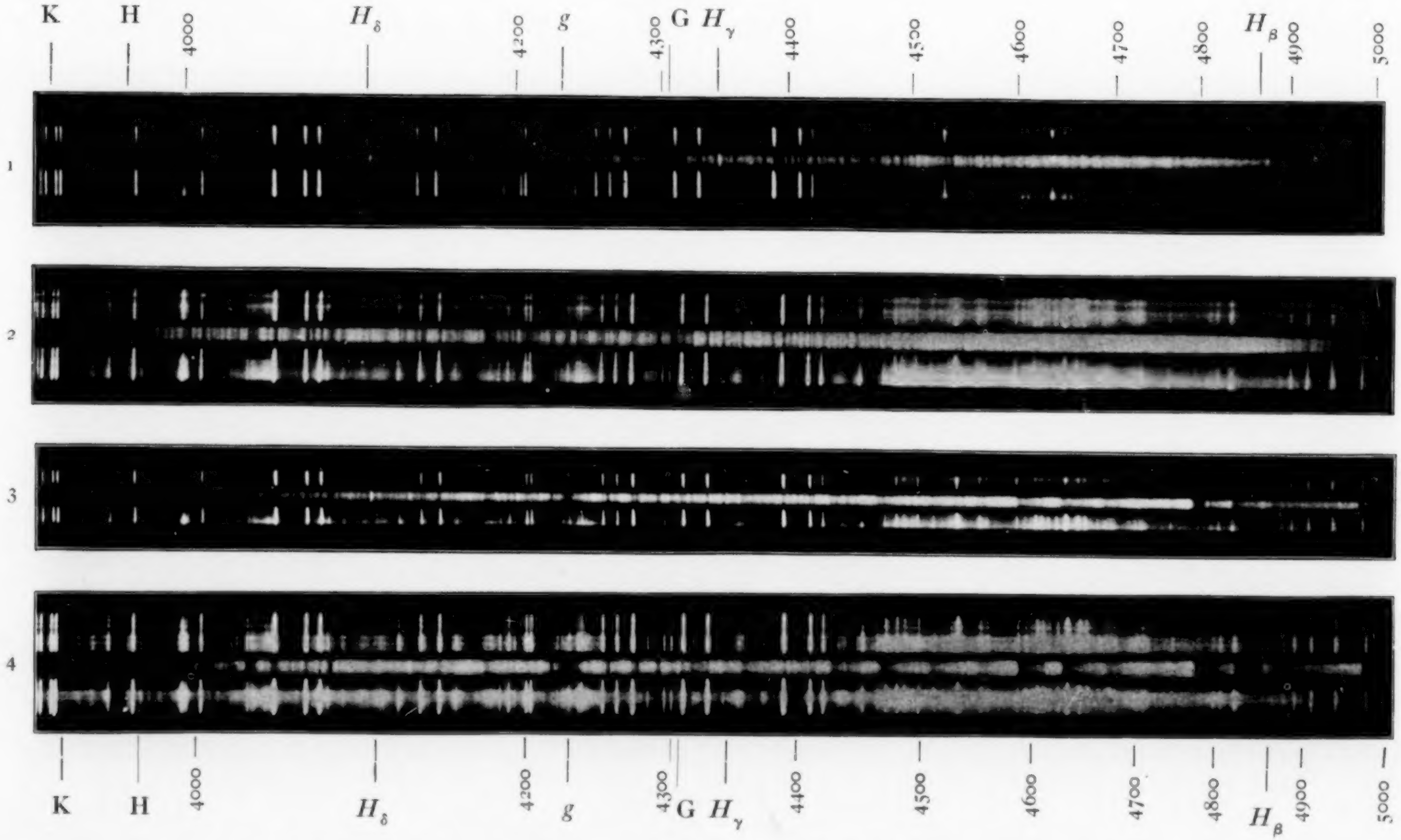
A detailed study of these plates will be made at the earliest opportunity.

RALPH H. CURTISS.

LICK OBSERVATORY,
July 1, 1904.

[1] Also to appear in a *Bulletin* of the Lick Observatory.

PLATE XIV.



1. *R Scuti*, 1903, July 19. 13 E.
2. *R Scuti*, 1903, September 21. 84 B.
3. *W Cygni*, 1903, August 3. 24 C.
4. *W Cygni*, 1903, October 12. 111 A.

SILICON LINES IN SPARK AND STELLAR SPECTRA.[1]

I have previously[2] shown that the spectral lines in the visible region of the oscillatory silicon spark may be divided into two groups, one of which includes those lines which are either unaffected or slightly enhanced by the introduction of inductance, while the other group includes those which disappear when inductance is used. These persistent lines of the first group, which I shall designate by *P*, have their origin in the "aureole" of the spark, and are doubtless due to a high temperature; those of the second group, which I shall designate by *D*, originate in the initial linear discharge, where it is probable that both dissociation and high temperature are at work.

By means of photography I have extended this research to the ultra-violet region and have compared these two groups with the corresponding lines found in stellar spectra by Lockyer[3] and Lunt.[4] The enhanced lines of silicon have been employed by Lockyer for making a tentative classification of stars on the basis of temperature; and my thought is that the effect of inductance upon these lines might shed some light upon this classification.

The spectra described below were obtained partly by use of two heavy flint prisms and partly by use of a single Rutherford prism. For purposes of comparison I have employed the ordinary spark spectrum of silicon, the spark-gap being shunted with a capacity of 0.009 microfarads. Alongside this spectrum was photographed that of the same spark after successive inductances, varying from 0.00002 to 0.03 henry, had been introduced. Wave-lengths were determined by comparison with those of a lead-cadmium alloy photographed on the same plate. The silicon employed was partly some which had been crystallized in small octohedra and plates, and partly sodium silicate cast in tubes about a platinum wire as core.

In the following table, the Roman numerals indicate the temperature groups of Lockyer, the numbers increasing with the temperature. The lines inclosed in parentheses coincide, to within errors of measurement, with air lines; they disappear also when the air lines disappear.

[1] Abridged translation of a paper appearing in *Comptes Rendus* (**139**, 188, July 18, 1904), sent by the author.

[2] *Comptes Rendus*, **134**, 1048, 1205, 1902.

[3] *Proc. R. S.*, **66**, 44, 1900.

[4] *Ibid.*, **67**, 403, 1901.

Lockyer	λ			Remarks
	α 6370.0	*P*	Strong.	Observed visually; should be sought in stellar spectra.
	6342.0	*P*	Strong.	
	β 5979.0	*D*	Easily visible.	
	5960.0	*D*	Easily visible.	
II	γ 5058.7	*P*	Strong.	Photographed on orthochromatic plates; should be sought in stellar spectra.
II	5044.0	*P*	Strong.	
III	δ 4574.6	*D*	Distinct.	*Orion* stars, *β Crucis*, *ϵ Canis Majoris;* weak in *α Cygni*.
III	4567.5	*D*	Strong.	
III	4552.3	*D*	Strong.	
II	ϵ 4131.0	*D*	Very strong, diffuse.	*Orion* stars, *Sirius*, *Procyon*, *Algol*, *α Cygni*.
II	4128.2	*D*	Very strong, diffuse.	
IV	(4116.5)	*D*	Short, very weak.	Stars of *Orion*, and *β Crucis*, where nitrogen lines have been recognized.
I	(4103.5)	*D*	Short, very weak.	Not found in stars of *Orion*, or in eclipse spectra; Rowland gives *Si* 4103.1 in Sun and arc.
IV	(4097.3)	*D*	Short, very weak.	*Orion*, *β Crucis*, *γ Argus*, where nitrogen and oxygen lines have been found.
IV	(4089.3)	*D*	Weak, diffuse.	
I	ζ_1 3905.7	*P*	Very strong, sharp.	Seen in arc, spark, Sun, and eclipse spectra, *Sirius*, *Polaris*, *Procyon*, *Aldebaran*, *Arcturus*.
II	ζ_2 3862.5	*D*	Strong, sharp.	*Orion* and *α Cygni*.
II	3856.2	*D*	Strong, sharp.	
	3854.0	....	Short, weak.	
	η 3807.5	*D*	Rather st'g, sharp.	*ϵ Canis Majoris*.
	3796.0	*D*	Rather st'g, sharp.	
	3791.5	*D*	Easily visible, sharp.	*Orion* stars.

By comparison with recent stellar photographs, the following conclusions appear to be warranted:

1. Only stars belonging to the first class show those lines which disappear under self-induction. Helium stars, such as those of *Orion* or *ϵ Canis Majoris*, give those lines which are the first to disappear, as, for instance, the triplet *Si* δ. The hydrogen stars such as *Sirius* and those which approach the solar type, as *Procyon*, yield those lines which are the last to disappear, *Si* ϵ and *Si* ζ_2.

2. Stars of the solar type exhibit those persistent lines which are common to arc and spark, *e. g.*, *Si* ζ_1; they are observed also in the flash spectrum. It will be interesting to determine whether the groups *Si* α and *Si* γ are not also found in stars of this class.

3. Stars of the third and fourth classes—presumably of low temperature—do not give any silicon lines.

It may be added that those lines which correspond to Lockyer's Group IV and which, according to him, indicate high temperature, are always associated on my plates with air lines; they coincide with lines of oxygen

and nitrogen, elements which have already been detected in several stars of the *Orion* type and in *β Crucis*.[1] I therefore consider group IV due to air.

As to the more refrangible part of the silicon spectrum, where no coincidences have been observed in stellar spectra, the strong line at λ 2542 disappears when the inductance reaches 0.006 henry.

The other lines, notably the characteristic group of six lines just below λ 2500 and the group at λ 2217–2209 show no diminution with my largest inductance, 0.03 henry.

A. DE GRAMONT.

PARIS, FRANCE,
July 18, 1904.

THE CROCKER ECLIPSE EXPEDITIONS IN 1905.[2]

THE next observable total solar eclipse occurs on August 30, 1905. It is remarkably well situated and is looked forward to with great interest. The shadow path begins at sunrise south of Hudson's Bay, enters the Atlantic Ocean a short distance north of Newfoundland, crosses northeastern Spain, northeastern Algiers and northern Tunis, passes centrally over Assuan on the Nile, and ends at sunset in northeastern Arabia. The durations on the coast of Labrador, in Spain, and at Assuan are two and one-half, three and three-fourths, and two and three-fifths minutes, respectively.

The interval of two hours and one-half between the instants of totality in Labrador and Egypt offers an unusual advantage for obtaining large-scale photographs of the solar corona, with a view to determining changes in the forms and positions of the delicate details of structure. The opportunity to bring the search for intra-mercurial planets to a satisfactory conclusion is also exceedingly promising. Should a new planet be observed at three stations, the interest attaching to its discovery would be heightened by the fact that its approximate orbit could be determined at once. If no planets are revealed on good photographs, the negative results would be scarcely less valuable, though certainly less interesting, than positive results, and the intra-mercurial question would cease to be a pressing eclipse problem.

An observing station in Spain would contribute to both of the above investigations, as well as to many polarigraphic and spectrographic studies.

An important element in the success of eclipse observations consists

[1] MCCLEAN, *Spectra of Southern Stars* (London, 1898), and "Comparative Photographic Spectra," *Phil. Transactions* (1898).

[2] From *Lick Observatory Bulletin* No. 59.

in the opportunity to prepare the instrumental equipment and the program and methods of observation well in advance, in order that critical tests may be made before the expeditions depart for their observing stations.

It is a pleasure to announce that Mr. William H. Crocker has again shown his interest in the science of astronomy by offering to meet the expenses of expeditions to be sent from the Lick Observatory, University of California, to Labrador, Spain, and Egypt, to secure observations of the 1905 eclipse.

The provisional program for the three stations is, in the main, as follows:

LABRADOR.

A photographic search for intra-mercurial planets in a region of the sky $8\frac{1}{2}°$ wide, extending along the direction of the solar equator from 4° below the Sun to 15° above it.

The photography of the corona by means of a camera of five inches aperture and forty-foot focus, of the form first used by Professor Schaeberle at the eclipse of 1893.

SPAIN.

A photographic intra-mercurial search covering a region $9\frac{1}{4}°$ wide, extending in the direction of the solar equator from 14° below to 14° above the Sun.

The photography of the solar corona with a camera of five inches aperture and forty-foot focus.

A polarigraphic study of the polarized light in the corona.

The use of spectrographs provided with moving plate-holders, to obtain continuous records of changes in the spectrum of the Sun's edge at the times of second and third contacts; of spectrographs for determining the wave-length of the green coronal bright line, and, if possible, the accurate wave-lengths of the bright and dark lines in the isolated spectrum of the Sun's edge, as nearly as possible at the time when the dark lines give way to bright ones, and *vice versa;* and of a spectrograph for recording the general spectrum of the corona.

EGYPT.

A photographic intra-mercurial search in a region $8\frac{1}{2}°$ wide, extending in the direction of the solar equator from 4° below to 15° above the Sun.

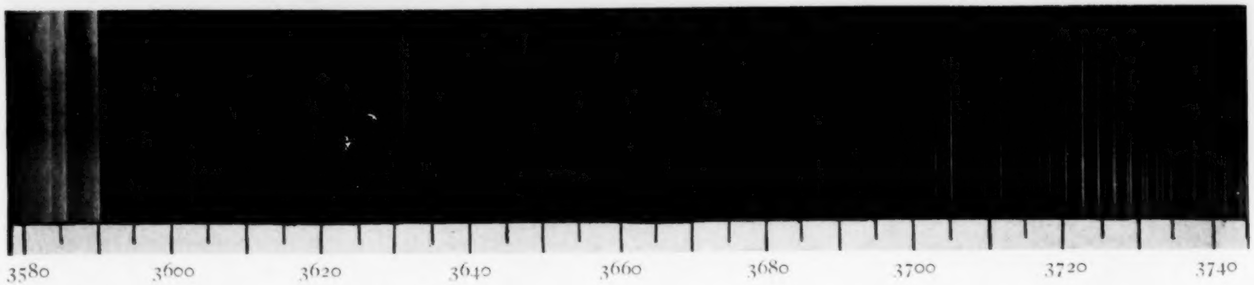
The photography of the solar corona with a camera of five inches aperture and forty-foot focus.

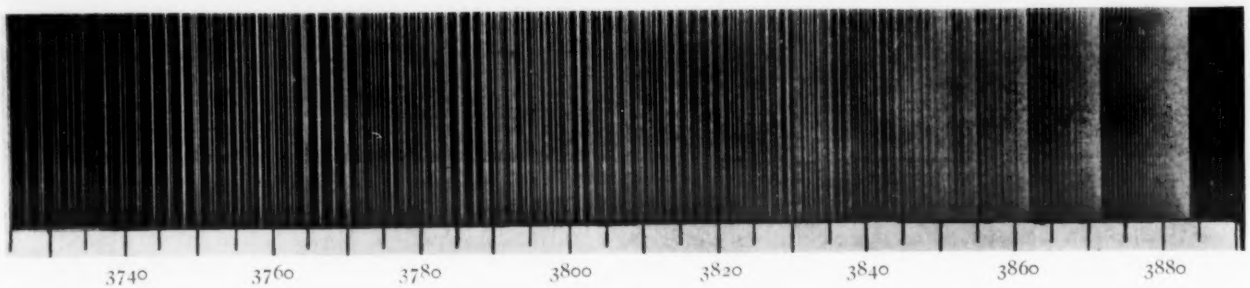
The photography of the general spectrum of the corona.

W. W. CAMPBELL.

MOUNT HAMILTON, CALIF.,
August 3, 1904.

PLATE XV.





THE THIRD CYANOGEN BAND.